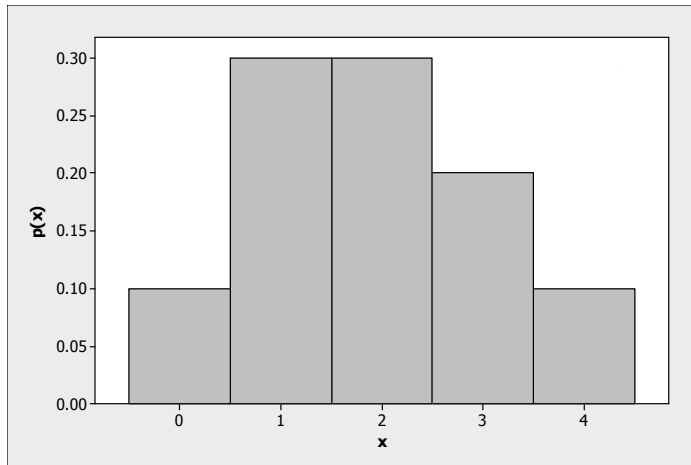


Exercices tirés du livre
Introduction to probability and statistics, second Canadian edition
de Mendenhall, Beaver, Beaver et Ahmed
STT1700
(Automne 2010)

Chapitre 3:

4.80, 4.81, 4.83, 4.85, 4.86, 4.89, 4.90, 4.95, 4.96, 4.98, 4.99, 4.102, 4.103, 4.106, 4.107, 4.114, 4.121, 4.122, 4.130, 5.3, 5.4, 5.20, 5.21, 5.22, 5.23, 5.25, 5.30, 5.33, 5.57, 5.66, 5.67, 5.70, 5.72.

- 4.80**
- a** The number of points scored is a discrete random variable taking the countably infinite number of values, 0, 1, 2, ...
 - b** Shelf life is a continuous random variable, since it can take on any positive real value.
 - c** Height is a continuous random variable, taking on any positive real value.
 - d** Length is a continuous random variable, taking on any positive real value.
 - e** Number of near collisions is a discrete random variable, taking the values 0, 1, 2, ...
- 4.81**
- a** The increase in length of life achieved by a cancer patient as a result of surgery is a continuous random variable, since an increase in life (measured in units of time) can take on any of an infinite number of values in a particular interval.
 - b** The tensile strength, in pounds per square inch, of one-inch diameter steel wire cable is a continuous random variable.
 - c** The number of deer killed per year in a state wildlife preserve is a discrete random variable taking the values 0, 1, 2, ...
 - d** The number of overdue accounts in a department store at a particular point in time is a discrete random variable, taking the values 0, 1, 2,
 - e** Blood pressure is a continuous random variable.
- 4.83**
- a** Since one of the requirements of a probability distribution is that $\sum_x p(x) = 1$, we need
$$p(3) = 1 - (.1 + .3 + .3 + .1) = 1 - .8 = .2$$
 - b** The probability histogram is shown below.



c For the random variable x given here, $\mu = E(x) = \sum xp(x) = 0(.1) + 1(.3) + \dots + 4(.1) = 1.9$

The variance of x is defined as

$$\sigma^2 = E[(x - \mu)^2] = \sum (x - \mu)^2 p(x) = (0 - 1.9)^2(.1) + (1 - 1.9)^2(.3) + \dots + (4 - 1.9)^2(.1) = 1.29$$

and $\sigma = \sqrt{1.29} = 1.136$.

d Using the table form of the probability distribution given in the exercise,

$$P(x > 2) = .2 + .1 = .3.$$

e $P(x \leq 3) = 1 - P(x = 4) = 1 - .1 = .9$.

4.85 For the probability distribution given in this exercise,

$$\mu = E(x) = \sum xp(x) = 0(.1) + 1(.4) + 2(.4) + 3(.1) = 1.5.$$

4.86 a Define D: person prefers David Letterman
J: person prefers Jay Leno

There are eight simple events in the experiment:

DDD DDJ
DJJ DJD
JDJ JDD
JJD JJJ

and the probabilities for x = number who prefer Jay Leno = 0, 1, 2, 3 are shown below.

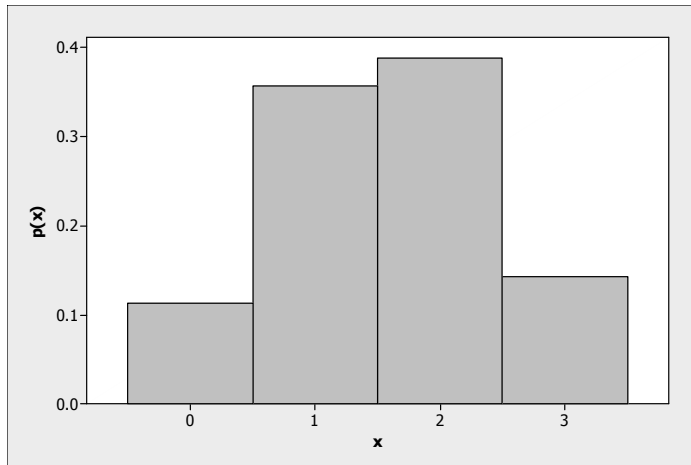
$$P(x = 0) = P(DDD) = (.48)^3 = .1106$$

$$P(x = 1) = P(DDJ) + P(DJD) + P(JDD) = 3(.52)(.48)^2 = .3594$$

$$P(x = 2) = P(DJJ) + P(JJD) + P(JDJ) = 3(.52)^2(.48) = .3894$$

$$P(x = 3) = P(JJJ) = (.52)^3 = .1406$$

b The probability histogram is shown below.



c $P(x = 1) = .3594$

d The average value of x is

$$\mu = E(x) = \sum xp(x) = 0(.1106) + 1(.3594) + 2(.3894) + 3(.1406) = 1.56$$

The variance of x is

$$\begin{aligned} \sigma^2 &= E[(x - \mu)^2] = \sum (x - \mu)^2 p(x) \\ &= (0 - 1.56)^2 (.1106) + (1 - 1.56)^2 (.3594) + (2 - 1.56)^2 (.3894) + (3 - 1.56)^2 (.1406) \\ &= .7488 \end{aligned}$$

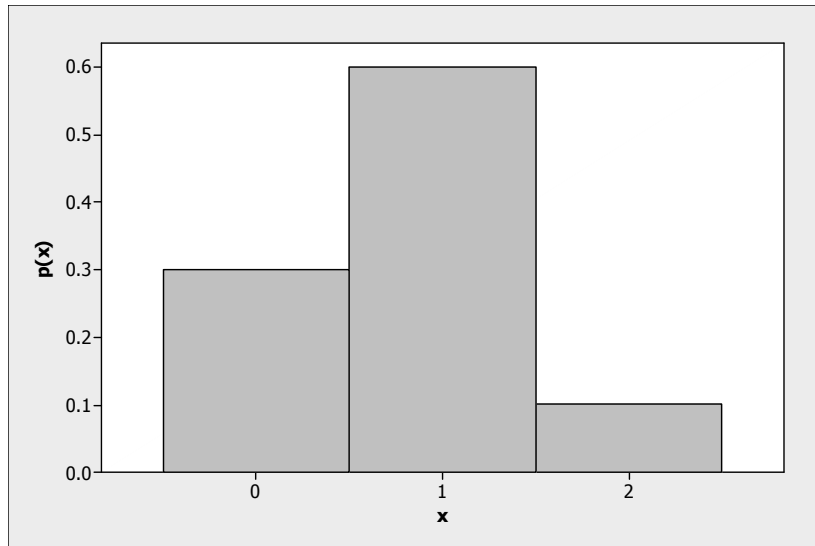
and $\sigma = \sqrt{.7488} = .865$.

4.89 a-b Let W_1 and W_2 be the two women while M_1 , M_2 and M_3 are the three men. There are 10 ways to

choose the two people to fill the positions. Let x be the number of women chosen. The 10 equally likely simple events are:

- | | |
|---------------------------|------------------------------|
| $E_1: W_1W_2$ ($x = 2$) | $E_6: W_2M_2$ ($x = 1$) |
| $E_2: W_1M_1$ ($x = 1$) | $E_7: W_2M_3$ ($x = 1$) |
| $E_3: W_1M_2$ ($x = 1$) | $E_8: M_1M_2$ ($x = 0$) |
| $E_4: W_1M_3$ ($x = 1$) | $E_9: M_1M_3$ ($x = 0$) |
| $E_5: W_2M_1$ ($x = 1$) | $E_{10}: M_2M_3$ ($x = 0$) |

The probability distribution for x is then $p(0) = 3/10$, $p(1) = 6/10$, $p(2) = 1/10$. The probability histogram is shown below.



- 4.90** Similar to Exercise 4.89. The random variable x can take on the values 0, 1, or 2. The associated probabilities can be found by summing probabilities of the simple events for the respective numerical events or by using the laws of probability

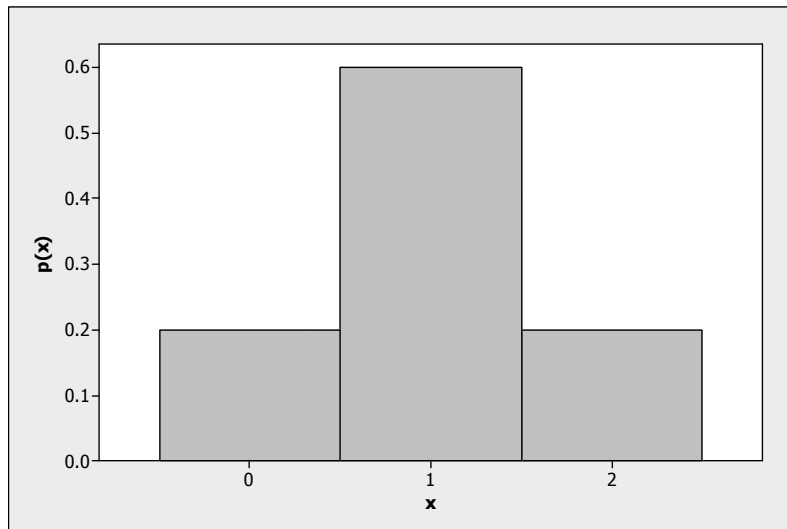
$$P[x=0] = P[\text{nondefective on first selection}]P[\text{nondefective on second} | \text{nondefective on first}] \\ \times P[\text{nondefective on third} | \text{nondefective on first and second}] = \frac{4}{6}\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) = \frac{1}{5}$$

$$P[x=1] = P(DNN) + P(NDN) + P(NND) = \frac{2}{6}\left(\frac{4}{5}\right)\left(\frac{3}{4}\right) + \frac{4}{6}\left(\frac{2}{5}\right)\left(\frac{3}{4}\right) + \frac{4}{6}\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) = \frac{3}{5}$$

$$P[x=2] = P(DDN) + P(DND) + P(NDD) = \frac{2}{6}\left(\frac{1}{5}\right)\left(\frac{4}{4}\right) + \frac{2}{6}\left(\frac{4}{5}\right)\left(\frac{1}{4}\right) + \frac{4}{6}\left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{5}$$

The probability distribution for x and the probability histogram follow.

x	0	1	2
$p(x)$	1/5	3/5	1/5



- 4.95** The random variable G , total gain to the insurance company, will be D if there is no theft, but $D - 50,000$ if there is a theft during a given year. These two events will occur with probability .99 and .01, respectively. Hence, the probability distribution for G is given below.

G	$p(G)$
D	.99
$D - 50,000$.01

The expected gain is

$$E(G) = \sum Gp(G) = .99D + .01(D - 50,000) \\ = D - 50,000$$

In order that $E(G) = 1000$, it is necessary to have $1000 = D - 500$ or $D = \$1500$.

- 4.96**
- a $\mu = E(x) = \sum xp(x) = 3(.03) + 4(.05) + \dots + 13(.01) = 7.9$
- b $\sigma^2 = \sum (x - \mu)^2 p(x) = (3 - 7.9)^2 (.03) + (4 - 7.9)^2 (.05) + \dots + (13 - 7.9)^2 (.01) = 4.73$ and $\sigma = \sqrt{4.73} = 2.1749$.
- c Calculate $\mu \pm 2\sigma = 7.9 \pm 4.350$ or 3.55 to 12.25. Then, referring to the probability distribution of x , $P[3.55 \leq x \leq 12.25] = P[4 \leq x \leq 12] = 1 - p(3) - p(13) = 1 - .04 = .96$.

- 4.98** The company will either gain $(\$15.50 - 14.80)$ if the package is delivered on time, or will lose $\$14.80$ if the package is not delivered on time. We assume that, if the package is not delivered within 24 hours, the company does not collect the $\$15.50$ delivery fee. Then the probability distribution for x , the company's gain is

x	$p(x)$	and
.70	.98	$\mu = E(x) = .70(.98) - 14.80(.02) = .39$.
-14.80	.02	

The expected gain per package is $\$0.39$.

- 4.99** We are asked to find the premium that the insurance company should charge in order to break even. Let c be the unknown value of the premium and x be the gain to the insurance company caused by marketing the new product. There are three possible values for x . If the product is a failure or moderately successful, x will be negative; if the product is a success, the insurance company will gain the amount of the premium and x will be positive. The probability distribution for x follows:

x	$p(x)$
c	.94
$-80,000 + c$.01
$-25,000 + c$.05

In order to break even, $E(x) = \sum xp(x) = 0$

$$.94(c) + .01(-80,000 + c) + (.05)(-25,000 + c) = 0$$

Therefore, $-800 - 1250 + (.01 + .05 + .94)c = 0$

$$c = 2050$$

Hence, the insurance company should charge a premium of $\$2,050$.

- 4.102 a** This experiment consists of two patients, each swallowing one of four tablets (two cold and two

aspirin). There are four tablets to choose from, call them C_1, C_2, A_1 and A_2 . The resulting simple events are then all possible ordered pairs which can be formed from the four choices.

(C_1C_2) (C_2C_1) (A_1C_1) (A_2C_1)
 (C_1A_1) (C_2A_1) (A_1C_2) (A_2C_2)
 (C_1A_2) (C_2A_2) (A_1A_2) (A_2A_1)

Notice that it is important to consider the order in which the tablets are chosen, since it makes a difference, for example, which patient (A or B) swallows the cold tablet.

- b** $A = \{(C_1C_2), (C_1A_1), (C_1A_2), (C_2A_1), (C_2C_1), (C_2A_2)\}$
- c** $B = \{(C_1A_1), (C_1A_2), (C_2A_1), (C_2A_2), (A_1C_1), (A_1C_2), (A_2C_1), (A_2C_2)\}$
- d** $C = \{(A_1A_2), (A_2A_1)\}$

4.103 Refer to Exercise 4.102. There are 12 simple events, each with equal probability $1/12$. By summing the probabilities of simple events in the events of interest we have

$$\begin{aligned} P(A) &= 6/12 = 1/2 & P(A \cup B) &= 10/12 = 5/6 \\ P(B) &= 8/12 = 2/3 & P(C) &= 2/12 = 1/6 \\ P(A \cap B) &= 4/12 = 1/3 & P(A \cap C) &= 0 \\ P(A \cup C) &= 8/12 = 2/3 \end{aligned}$$

4.106 Define the random variable x to be daily sales; x can take the values \$0, 50,000 or 100,000 depending on the number of customers the salesman contacts. The associated probabilities are shown below:

$$\begin{aligned} P[x = 0] &= P[\text{contact one, fail to sell}] + P[\text{contact two, fail to sell}] \\ &= P[\text{contact one}]P[\text{fail to sell}] + P[\text{contact two}]P[\text{fail with first}]P[\text{fail with second}] \\ &= (1/3)(9/10) + (2/3)(9/10)(9/10) \\ &= 9/30 + 162/300 = 252/300 \end{aligned}$$

Similarly,

$$\begin{aligned} P[x = 50,000] &= P[\text{contact one, sell}] + P[\text{contact two, sell to one}] \\ &= P[\text{contact one, sell}] + P[\text{contact two, sell to first only}] + P[\text{contact two, sell to second only}] \\ &= (1/3)(1/10) + (2/3)(1/10)(9/10) + (2/3)(9/10)(1/10) = 46/300 \end{aligned}$$

$$\begin{aligned} \text{Finally, } P[x = 100,000] &= P[\text{contact two, sell to both}] \\ &= (2/3)(1/10)(1/10) = 2/300 \end{aligned}$$

Then $E(x) = 0(252/300) + 50,000(46/300) + 100,000(2/300) = 8333.33$. Thus, the expected value of daily sales is \$8333.33.

4.107 The random variable x , defined as the number of householders insured against fire, can assume the values 0, 1, 2, 3 or 4. The probability that, on any of the four draws, an insured person is found is .6; hence, the probability of finding an uninsured person is .4. Note that each numerical event represents the intersection of the results of four independent draws.

a $P[x = 0] = (.4)(.4)(.4)(.4) = .0256$, since all four people must be uninsured.

b $P[x = 1] = 4(.6)(.4)(.4)(.4) = .1536$ (Note: the 4 appears in this expression because $x = 1$ is the union of four mutually exclusive events. These represent the 4 ways to choose the single insured person from the fours.)

c $P[x = 2] = 6(.6)(.6)(.4)(.4) = .3456$, since the two insured people can be chosen in any of 6 ways.

d $P[x = 3] = 4(.6)^3(.4) = .3456$ and $P[x = 4] = (.6)^4 = .1296$.

Then $P[\text{at least three insured}] = p(3) + p(4) = .3456 + .1296 = .4752$

4.114 Define the following events:
 B: man takes the bus
 S: man takes the subway
 L: the man is late

It is given that $P(B) = .3$, $P(S) = .7$, $P(L|B) = .3$, $P(L|S) = .2$. Using Bayes' Rule,

$$P(B|L) = \frac{P(L|B)P(B)}{P(L|B)P(B) + P(L|S)P(S)} = \frac{(.3)(.3)}{(.3)(.3) + (.2)(.7)} = \frac{.09}{.23} = .3913$$

4.121 Define R: the employee remains 10 years or more

a The probability that the man will stay less than 10 years is $P(R^c) = 1 - P(R) = 1 - 1/6 = 5/6$

b The probability that the man and the woman, acting independently, will both work less than 10 years is $P(R^c R^c) = P(R^c)P(R^c) = (5/6)^2 = 25/36$

c The probability that either or both people work 10 years or more is $1 - P(R^c R^c) = 1 - (5/6)^2 = 1 - 25/36 = 11/36$

4.122 Let y represent the value of the premium which the insurance company charges and let x be the insurance company's gain. There are four possible values for x . If no accident occurs or if an accident results in no damage to the car, the insurance company gains y dollars. If an accident occurs and the car is damaged, the company will gain either $y - 22,000$ dollars, $y - .6(22,000)$ dollars, or $y - .2(22,000)$ dollars, depending upon whether the damage to the car is total, 60% of market value, or 20% of market value, respectively. The following probabilities are known.

$$P[\text{accident occurs}] = .15 \qquad P[\text{total loss} \mid \text{accident occurs}] = .08$$

$$P[60\% \text{ loss} \mid \text{accident occurs}] = .12 \qquad P[20\% \text{ loss} \mid \text{accident occurs}] = .80$$

Hence,

$$P[x = y - 22,000] = P[\text{accident}]P[\text{total loss} \mid \text{accident}] = .15(.08) = .012$$

Similarly,

$$P[x = y - 13,200] = .15(.12) = .018 \text{ and } P[x = y - 4400] = .15(.80) = .12$$

The gain x and its associated probability distribution are shown below. Note that $p(y)$ is found by subtraction.

x	$p(x)$
$y - 22,000$.012
$y - 13,200$.018
$y - 4400$.12
y	.85

Letting the expected gain equal zero, the value of the premium is obtained.

$$E(x) = \sum xp(x) = .012(y - 22,000) + .018(y - 13,200) + .12(y - 4400) + .85y$$

$$E(x) = y - (264 + 237.6 + 528) = y - 1029.6$$

$$y = \$1029.6$$

4.130 a Define the event R: subject chooses red and N: subject does not choose red. Then

$P(R) = \frac{1}{3}$ and $P(N) = \frac{2}{3}$. There are 8 simple events in the experiment:

NNN ($x = 0$)	RRN ($x = 2$)
RNN ($x = 1$)	RNR ($x = 2$)
NRN ($x = 1$)	NRR ($x = 2$)
NNR ($x = 1$)	RRR ($x = 3$)

Then

$$P(x = 0) = P(NNN) = P(N)P(N)P(N) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$P(x = 1) = 3P(N)P(N)P(R) = 3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right) = \frac{12}{27}$$

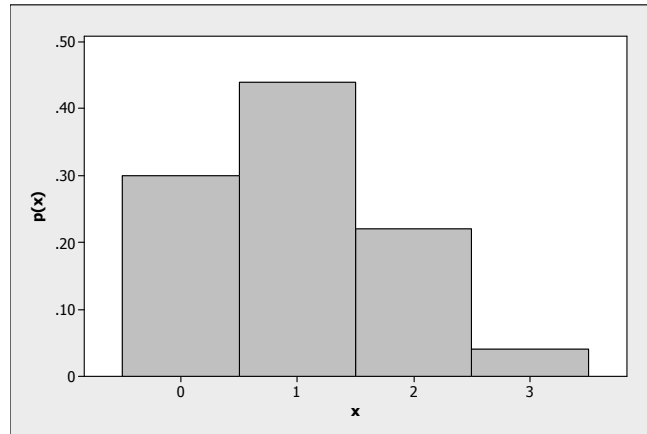
$$P(x = 2) = 3P(N)P(R)P(R) = 3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2 = \frac{6}{27}$$

$$P(x = 3) = P(RRR) = P(R)P(R)P(R) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

The probability distribution for x is shown in the table.

x	0	1	2	3
$p(x)$	8/27	12/27	6/27	1/27

b The probability histogram is shown below.



Chapter 5: Several Useful Discrete Distributions

k	0	1	2	3	4	5	6	7	8
$P(x \leq k)$.000	.001	.011	.058	.194	.448	.745	.942	1.000

5.3 The random variable x is not a binomial random variable since the balls are selected without replacement. For this reason

5.4 If the sampling in Exercise 5.3 is conducted with replacement, then x is a binomial random variable with $n = 2$ independent trials, and $p = P[\text{red ball}] = 3/5$, which remains constant from trial to trial.

5.20 Although there are $n = 30$ days on which it either rains (S) or does not rain (F), the random variable x would not be a binomial random variable because the trials (days) are *not independent*. If there is rain on one day, it will probably affect the probability that there will be rain on the next day.

5.21 Although there are trials (telephone calls) which result in either a person who will answer (S) or a person who will not (F), the number of trials, n , is not fixed in advance. Instead of recording x , the number of *successes* in n trials, you record x , the number of *trials* until the first success. This is *not* a binomial experiment.

5.22 There are $n = 100$ students which represent the experimental units.

- a** Each student either took (S) or did not take (F) the MCAT. Since the population of students is large, the probability $p = .45$ that a particular student took the MCAT will not vary from student to student and the trials will be independent. This is a binomial random variable.
- b** The measurement taken on each student is *score* which can take more than two values. This is not a binomial random variable.
- c** Each student either will (S) or will not (F) score above average. As in part **a**, the trials are independent although the value of p , the proportion of students in the population who score above average, is unknown. This is a binomial experiment.

- d** The measurement taken on each student is *amount of time* which can take more than two values. This is not a binomial random variable.
- 5.23** Define x to be the number of alarm systems that are triggered. Then $p = P[\text{alarm is triggered}] = .99$ and $n = 9$. Since there is a table available in Appendix I for $n = 9$ and $p = .99$, you should use it rather than the binomial formula to calculate the necessary probabilities.
- a** $P[\text{at least one alarm is triggered}] = P(x \geq 1) = 1 - P(x = 0) = 1 - .000 = 1.000$.
- b** $P[\text{more than seven}] = P(x > 7) = 1 - P(x \leq 7) = 1 - .003 = .997$
- c** $P[\text{eight or fewer}] = P(x \leq 8) = .086$
- 5.25** Define x to be the number of cars that are black. Then $p = P[\text{black}] = .1$ and $n = 25$. Use Table 1 in Appendix I.
- a** $P(x \geq 5) = 1 - P(x \leq 4) = 1 - .902 = .098$
- b** $P(x \leq 6) = .991$
- c** $P(x > 4) = 1 - P(x \leq 4) = 1 - .902 = .098$
- d** $P(x = 4) = P(x \leq 4) - P(x \leq 3) = .902 - .764 = .138$
- e** $P(3 \leq x \leq 5) = P(x \leq 5) - P(x \leq 2) = .967 - .537 = .430$
- f** $P(\text{more than 20 not black}) = P(\text{less than 5 black}) = P(x \leq 4) = .902$
- 5.30** Define x to be the number of times the mouse chooses the red door. Then, if the mouse actually has no preference for colour, $p = P[\text{red door}] = .5$ and $n = 10$. Since $\mu = np = 5$ and $\sigma = \sqrt{npq} = 1.58$, you would expect that, if there is no colour preference, the mouse should choose the red door
- $$\mu \pm 2\sigma \Rightarrow 5 \pm 3.16 \Rightarrow 1.84 \text{ to } 8.16$$
- or between 2 and 8 times. If the mouse chooses the red door more than 8 or less than 2 times, the unusual results might suggest a colour preference.
- 5.33** Define x to be the number of Canadians who are “tasters”. Then, $n = 20$ and $p = .7$. Using the binomial tables in Appendix I,
- a** $P(x \geq 17) = 1 - P(x \leq 16) = 1 - .893 = .107$
- b** $P(x \leq 15) = .762$
- 5.57** See Section 5.2 in the text.
- 5.66** Let x be the number of Canadians in the sample of $n = 15$ who are in favour of income splitting for couples. Then x has a binomial distribution with $p = .77 \approx .80$
- a** $p(x) = C_x^{15} (.8)^x (.2)^{15-x}$ for $x = 0, 1, 2, \dots, 15$.
- b** From Table 1, $P(x \leq 8) = .018$.
- c** $P(x > 8) = 1 - P(x \leq 8) = 1 - .018 = .982$.
- d** From Table 1 in Appendix I, look down the column for $p = .8$ to find the largest cumulative probability which is still less than or equal to .10. This probability is .061 with $P(x \leq 9) = .061$. Therefore, the largest value of c is $c = 9$.

- 5.67** Let x be the number of Canadians who rank owning a vacation home as the number one status symbol. Then x has a binomial distribution with $n = 400$ and $p = .6$.
- a** $\mu = np = 400(.6) = 240$
 - b** $\sigma = \sqrt{npq} = \sqrt{400(.6)(.4)} = 9.798$
 - c** Since n is large and $p = .6$, this binomial distribution will be relatively mound-shaped, and approximately 95% of the values of x should lie in the interval

$$\mu \pm 2\sigma \Rightarrow 240 \pm 19.596 \Rightarrow 220.404 \text{ to } 259.596$$
 - d** The value $x = 200$ lies $(200 - 240)/9.798 = -4.08$ standard deviations below the mean. This is an unusual result. Perhaps the sample was not randomly selected, or perhaps the 60% figure is no longer accurate, and in fact this percentage has decreased.

- 5.70** Define x to be the number of students favoring the issue, with $n = 25$ and $p = P[\text{student favors the issue}]$ assumed to be .8. Using the binomial tables in Appendix I, $P[x \leq 15] = .017$
- Thus, the probability of observing $x = 15$ or the more extreme values, $x = 0, 1, 2, \dots, 14$ is quite small under the assumption that $p = .8$. We probably should conclude that p is actually smaller than .8.

- 5.72**
- a** $p = P[\text{rain according to the forecaster}] = .3$
 - b** With $n = 25$ and $p = .3$, $\mu = np = 25(.3) = 7.5$ and $\sigma = \sqrt{npq} = \sqrt{5.25} = 2.29129$
 - c** The observed value, $x = 10$, lies $z = \frac{10 - 7.5}{2.29} = 1.09$ standard deviations above the mean.
 - d** The observed event is not unlikely under the assumption that $p = .3$. We have no reason to doubt the forecaster.