Exercices tirés du livre
Introduction to probability and statistics, second Canadian edition
de Mendenhall, Beaver, Beaver et Ahmed
STT1700
(Automne 2010)

Chapitre 3:

4.80, 4.81, 4.83, 4.85, 4.86, 4.89, 4.90, 4.95, 4.96, 4.98, 4.99, 4.102, 4.103, 4.106, 4.107, 4.114, 4.121, 4.122, 4.130, 5.3, 5.4, 5.20, 5.21, 5.22, 5.23, 5.25, 5.30, 5.33, 5.57, 5.66, 5.67, 5.70, 5.72.

4.80 **a** The number of points scored is a discrete random variable taking the countably infinite number of values, 0, 1, 2, …

b Shelf life is a continuous random variable, since it can take on any positive real value.

c Height is a continuous random variable, taking on any positive real value.

d Length is a continuous random variable, taking on any positive real value.

e Number of near collisions is a discrete random variable, taking the values 0, 1, 2, …

4.81 **a** The increase in length of life achieved by a cancer patient as a result of surgery is a continuous random variable, since an increase in life (measured in units of time) can take on any of an infinite number of values in a particular interval.

b The tensile strength, in pounds per square inch, of one-inch diameter steel wire cable is a continuous random variable.

c The number of deer killed per year in a state wildlife preserve is a discrete random variable taking the values 0,1, 2, …

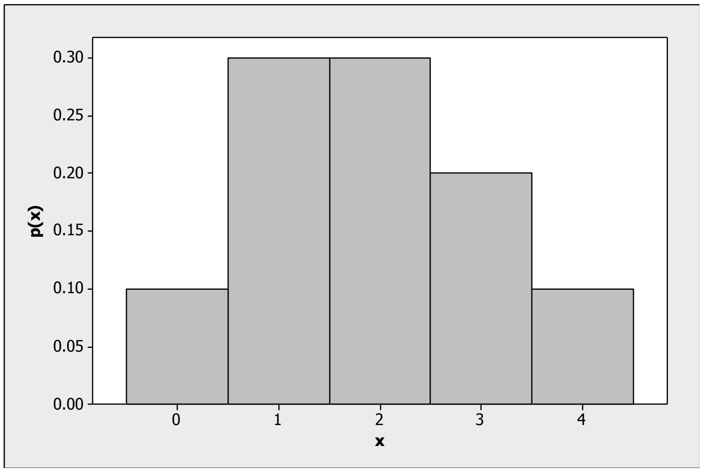
d The number of overdue accounts in a department store at a particular point in time is a discrete random variable, taking the values 0, 1, 2, ….

e Blood pressure is a continuous random variable.

4.83 **a** Since one of the requirements of a probability distribution is that $\sum_x p(x) = 1$, we need

$$p(3) = 1 - (.1 + .3 + .3 + .1) = 1 - .8 = .2$$

b The probability histogram is shown below.



c For the random variable x given here, $\mu = E(x) = \sum xp(x) = 0(.1) + 1(.3) + \cdots + 4(.1) = 1.9$

The variance of x is defined as

$$\sigma^2 = E\left[(x-\mu)^2\right] = \sum (x-\mu)^2 p(x) = (0-1.9)^2(.1) + (1-1.9)^2(.3) + \cdots + (4-1.9)^2(.1) = 1.29$$

and $\sigma = \sqrt{1.29} = 1.136$.

d Using the table form of the probability distribution given in the exercise, $P(x > 2) = .2 + .1 = .3$.

e $P(x \le 3) = 1 - P(x = 4) = 1 - .1 = .9$.

4.85 For the probability distribution given in this exercise,

$$\mu = E(x) = \sum xp(x) = 0(.1) + 1(.4) + 2(.4) + 3(.1) = 1.5.$$

4.86 **a** Define D: person prefers David Letterman

J: person prefers Jay Leno

There are eight simple events in the experiment:

DDD	DDJ
DJJ	DJD
JDJ	JDD
JJD	JJJ

and the probabilities for x = number who prefer Jay Leno = 0, 1, 2, 3 are shown below.

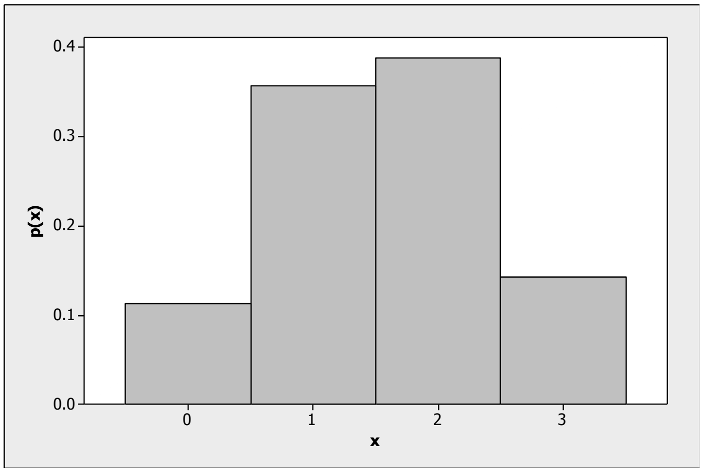
$$P(x=0) = P(DDD) = (.48)^3 = .1106$$

$$P(x=1) = P(DDJ) + P(DJD) + P(JDD) = 3(.52)(.48)^2 = .3594$$

$$P(x=2) = P(DJJ) + P(JJD) + P(JDJ) = 3(.52)^2(.48) = .3894$$

$$P(x=3) = P(JJJ) = (.52)^3 = .1406$$

b The probability histogram is shown below.



c $P(x=1)=.3594$

d The average value of x is

$$\mu = E(x) = \sum xp(x) = 0(.1106)+1(.3594)+2(.3894)+3(.1406) = 1.56$$

The variance of x is

$$\sigma^2 = E\left[(x-\mu)^2\right] = \sum (x-\mu)^2 p(x)$$

$$= (0-1.56)^2(.1106)+(1-1.56)^2(.3594)+(2-1.56)^2(.3894)+(3-1.56)^2(.1406)$$

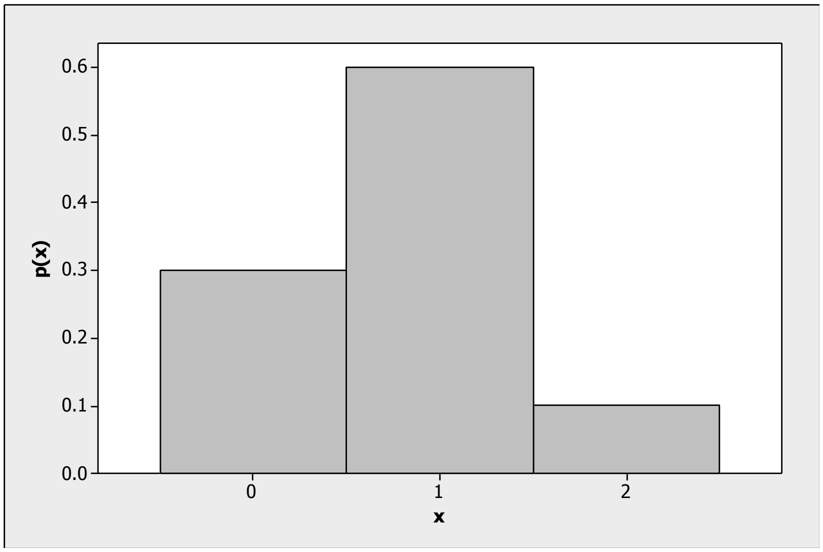
$$= .7488$$

and $\sigma = \sqrt{.7488} = .865$.

4.89 **a-b** Let W_1 and W_2 be the two women while M_1, M_2 and M_3 are the three men. There are 10 ways to choose the two people to fill the positions. Let x be the number of women chosen. The 10 equally likely simple events are:

E_1: W_1W_2 $(x=2)$	E_6: W_2M_2 $(x=1)$
E_2: W_1M_1 $(x=1)$	E_7: W_2M_3 $(x=1)$
E_3: W_1M_2 $(x=1)$	E_8: M_1M_2 $(x=0)$
E_4: W_1M_3 $(x=1)$	E_9: M_1M_3 $(x=0)$
E_5: W_2M_1 $(x=1)$	E_{10}: M_2M_3 $(x=0)$

The probability distribution for x is then $p(0)=3/10$, $p(1)=6/10$, $p(2)=1/10$. The probability histogram is shown below.



0.6
0.5
0.4
0.3
0.2
0.1
0.0
p(x)
0
1
2
x

4.90 Similar to Exercise 4.89. The random variable *x* can take on the values 0, 1, or 2. The associated probabilities can be found by summing probabilities of the simple events for the respective numerical events or by using the laws of probability

$$P[x=0] = P[\text{nondefective on first selection}]P[\text{nondefective on second} \mid \text{nondefective on first}]$$

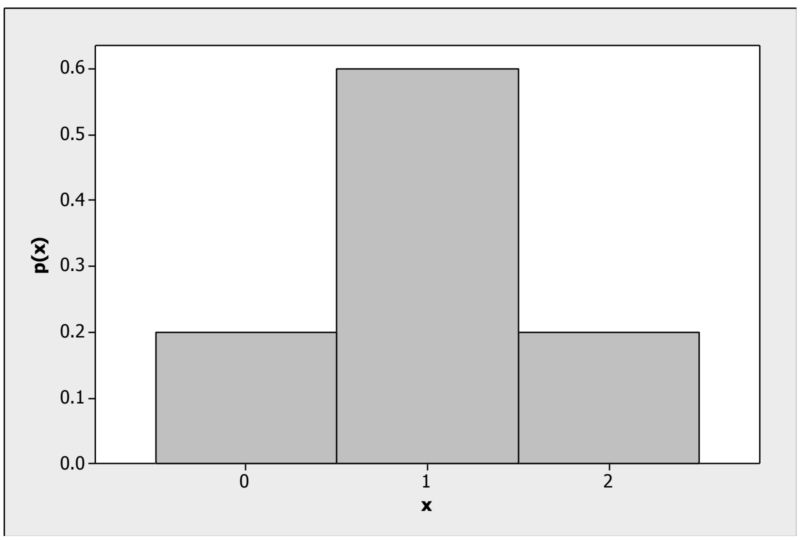
$$\times P[\text{nondefective on third} \mid \text{nondefective on first and second}] = \frac{4}{6}\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) = \frac{1}{5}$$

$$P[x=1] = P(DNN) + P(NDN) + P(NND) = \frac{2}{6}\left(\frac{4}{5}\right)\left(\frac{3}{4}\right) + \frac{4}{6}\left(\frac{2}{5}\right)\left(\frac{3}{4}\right) + \frac{4}{6}\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) = \frac{3}{5}$$

$$P[x=2] = P(DDN) + P(DND) + P(NDD) = \frac{2}{6}\left(\frac{1}{5}\right)\left(\frac{4}{4}\right) + \frac{2}{6}\left(\frac{4}{5}\right)\left(\frac{1}{4}\right) + \frac{4}{6}\left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{5}$$

The probability distribution for *x* and the probability histogram follow.

x	0	1	2
p(x)	1/5	3/5	1/5



4.95 The random variable G, total gain to the insurance company, will be D if there is no theft, but D – 50,000 if there is a theft during a given year. These two events will occur with probability .99 and .01, respectively. Hence, the probability distribution for G is given below.

G	*p*(G)
D	.99
D – 50,000	.01

The expected gain is

$$E(G) = \sum Gp(G) = .99D + .01(D - 50{,}000)$$
$$= D - 50{,}000$$

In order that $E(G) = 1000$, it is necessary to have $1000 = D - 500$ or $D = \$1500$.

4.96 **a** $\mu = E(x) = \sum xp(x) = 3(.03) + 4(.05) + \cdots + 13(.01) = 7.9$

b $\sigma^2 = \sum (x-\mu)^2\, p(x) = (3-7.9)^2(.03) + (4-7.9)^2(.05) + \cdots + (13-7.9)^2(.01) = 4.73$ and $\sigma = \sqrt{4.73} = 2.1749$.

c Calculate $\mu \pm 2\sigma = 7.9 \pm 4.350$ or 3.55 to 12.25. Then, referring to the probability distribution of *x*, $P[3.55 \le x \le 12.25] = P[4 \le x \le 12] = 1 - p(3) - p(13) = 1 - .04 = .96$.

4.98 The company will either gain ($15.50 – 14.80) if the package is delivered on time, or will lose $14.80 if the package is not delivered on time. We assume that, if the package is not delivered within 24 hours, the company does not collect the $15.50 delivery fee. Then the probability distribution for *x*, the company's gain is

x	$p(x)$
.70	.98
–14.80	.02

and

$$\mu = E(x) = .70(.98) - 14.80(.02) = .39 .$$

The expected gain per package is $0.39.

4.99 We are asked to find the premium that the insurance company should charge in order to break even. Let *c* be the unknown value of the premium and *x* be the gain to the insurance company caused by marketing the new product. There are three possible values for *x*. If the product is a failure or moderately successful, *x* will be negative; if the product is a success, the insurance company will gain the amount of the premium and *x* will be positive. The probability distribution for *x* follows:

x	$p(x)$
c	.94
$-80{,}000 + c$	.01
$-25{,}000 + c$	.05

In order to break even, $E(x) = \sum xp(x) = 0$

$$.94(c) + .01(-80,000 + c) + (.05)(-25,000 + c) = 0$$

Therefore, $$-800 - 1250 + (.01 + .05 + .94)c = 0$$

$$c = 2050$$

Hence, the insurance company should charge a premium of $2,050.

4.102 **a** This experiment consists of two patients, each swallowing one of four tablets (two cold and two aspirin). There are four tablets to choose from, call them C_1, C_2, A_1 and A_2. The resulting simple events are then all possible ordered pairs which can be formed from the four choices.

(C_1C_2) (C_2C_1) (A_1C_1) (A_2C_1)
(C_1A_1) (C_2A_1) (A_1C_2) (A_2C_2)
(C_1A_2) (C_2A_2) (A_1A_2) (A_2A_1)

Notice that it is important to consider the order in which the tablets are chosen, since it makes a difference, for example, which patient (A or B) swallows the cold tablet.

b $A = \{(C_1C_2),(C_1A_1),(C_1A_2),(C_2A_1),(C_2C_1),(C_2A_2)\}$

c $B = \{(C_1A_1),(C_1A_2),(C_2A_1),(C_2A_2),(A_1C_1),(A_1C_2),(A_2C_1),(A_2C_2)\}$

d $C = \{(A_1A_2),(A_2A_1)\}$

4.103 Refer to Exercise 4.102. There are 12 simple events, each with equal probability 1/12. By summing the probabilities of simple events in the events of interest we have

$P(A) = 6/12 = 1/2$ $\quad$ $P(A \cup B) = 10/12 = 5/6$

$P(B) = 8/12 = 2/3$ $\quad$ $P(C) = 2/12 = 1/6$

$P(A \cap B) = 4/12 = 1/3$ $\quad$ $P(A \cap C) = 0$

$P(A \cup C) = 8/12 = 2/3$

4.106 Define the random variable x to be daily sales; x can take the values \$0, 50,000 or 100,000 depending on the number of customers the salesman contacts. The associated probabilities are shown below:

$$\begin{aligned} P[x=0] &= P[\text{contact one, fail to sell}] + P[\text{contact two, fail to sell}] \\ &= P[\text{contact one}]P[\text{fail to sell}] + P[\text{contact two}]P[\text{fail with first}]P[\text{fail with second}] \\ &= (1/3)(9/10) + (2/3)(9/10)(9/10) \\ &= 9/30 + 162/300 = 252/300 \end{aligned}$$

Similarly,

$$\begin{aligned} P[x=50,000] &= P[\text{contact one, sell}] + P[\text{contact two, sell to one}] \\ &= P[\text{contact one, sell}] + P[\text{contact two, sell to first only}] + P[\text{contact two, sell to second only}] \\ &= (1/3)(1/10) + (2/3)(1/10)(9/10) + (2/3)(9/10)(1/10) = 46/300 \end{aligned}$$

Finally, $P[x=100,000] = P[\text{contact two, sell to both}]$

$$= (2/3)(1/10)(1/10) = 2/300$$

Then $E(x) = 0(252/300) + 50,000(46/300) + 100,000(2/300) = 8333.33$. Thus, the expected value of daily sales is \$8333.33.

4.107 The random variable *x*, defined as the number of householders insured against fire, can assume the values 0, 1, 2, 3 or 4. The probability that, on any of the four draws, an insured person is found is .6; hence, the probability of finding an uninsured person is .4. Note that each numerical event represents the intersection of the results of four independent draws.

a $P[x=0]=(.4)(.4)(.4)(.4)=.0256$, since all four people must be uninsured.

b $P[x=1]=4(.6)(.4)(.4)(.4)=.1536$ (Note: the 4 appears in this expression because $x=1$ is the union of four mutually exclusive events. These represent the 4 ways to choose the single insured person from the fours.)

c $P[x=2]=6(.6)(.6)(.4)(.4)=.3456$, since the two insured people can be chosen in any of 6 ways.

d $P[x=3]=4(.6)^3(.4)=.3456$ and $P[x=4]=(.6)^4=.1296$.

Then $P[\text{at least three insured}]=p(3)+p(4)=.3456+.1296=.4752$

4.114 Define the following events:
B: man takes the bus
S: man takes the subway
L: the man is late

It is given that $P(B)=.3,\ P(S)=.7,\ P(L\,|\,B)=.3,\ P(L\,|\,S)=.2$. Using Bayes' Rule,

$$P(B\,|\,L)=\frac{P(L\,|\,B)P(B)}{P(L\,|\,B)P(B)+P(L\,|\,S)P(S)}=\frac{(.3)(.3)}{(.3)(.3)+(.2)(.7)}=\frac{.09}{.23}=.3913$$

4.121 Define R: the employee remains 10 years or more

a The probability that the man will stay less than 10 years is $P(R^C)=1-P(R)=1-1/6=5/6$

b The probability that the man and the woman, acting independently, will both work less than 10 years is $P\left(R^C R^C\right)=P\left(R^C\right)P\left(R^C\right)=(5/6)^2=25/36$

c The probability that either or both people work 10 years or more is

$$1-P\left(R^C R^C\right)=1-(5/6)^2=1-25/36=11/36$$

4.122 Let *y* represent the value of the premium which the insurance company charges and let *x* be the insurance company's gain. There are four possible values for *x*. If no accident occurs or if an accident results in no damage to the car, the insurance company gains *y* dollars. If an accident occurs and the car is damaged, the company will gain either $y-22,000$ dollars, $y-.6(22,000)$ dollars, or $y-.2(22,000)$ dollars, depending upon whether the damage to the car is total, 60% of market value, or 20% of market value, respectively. The following probabilities are known.

$P[\text{accident occurs}]=.15$ $\quad$ $P[\text{total loss | accident occurs}]=.08$

$P[\text{60\% loss | accident occurs}]=.12$ $\quad$ $P[\text{20\% loss | accident occurs}]=.80$

Hence,

$$P[x=y-22,000]=P[\text{accident}]P[\text{total loss | accident}]=.15(.08)=.012$$

Similarly,

$$P[x=y-13,200]=.15(.12)=.018 \text{ and } P[x=y-4400]=.15(.80)=.12$$

The gain *x* and its associated probability distribution are shown below. Note that *p*(*y*) is found by subtraction.

x	*p*(*x*)
$y-22,000$	.012
$y-13,200$	.018
$y-4400$	.12
y	.85

Letting the expected gain equal zero, the value of the premium is obtained.

$$E(x)=\sum xp(x)=.012(y-22,000)+.018(y-13,200)+.12(y-4400)+.85y$$

$$E(x)=y-(264+237.6+528)=y-1029.6$$

$$y=\$1029.6$$

4.130 **a** Define the event R: subject chooses red and N: subject does not choose red. Then $P(R)=\frac{1}{3}$ and $P(N)=\frac{2}{3}$. There are 8 simple events in the experiment:

NNN $(x=0)$	RRN $(x=2)$
RNN $(x=1)$	RNR $(x=2)$
NRN $(x=1)$	NRR $(x=2)$
NNR $(x=1)$	RRR $(x=3)$

Then

$$P(x=0)=P(NNN)=P(N)P(N)P(N)=\left(\frac{2}{3}\right)^3=\frac{8}{27}$$

$$P(x=1)=3P(N)P(N)P(R)=3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)=\frac{12}{27}$$

$$P(x=2)=3P(N)P(R)P(R)=3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2=\frac{6}{27}$$

$$P(x=3)=P(RRR)=P(R)P(R)P(R)=\left(\frac{1}{3}\right)^3=\frac{1}{27}$$

The probability distribution for *x* is shown in the table.

x	0	1	2	3
$p(x)$	8/27	12/27	6/27	1/27

b The probability histogram is shown below.