8: Large-Sample Estimation

8.1 The margin of error in estimation provides a practical upper bound to the difference between a particular estimate and the parameter which it estimates. In this chapter, the margin of error is $1.96\times$ (standard error of the estimator).

8.3 For the estimate of μ given as $\overline{x}$, the margin of error is $1.96\ SE = 1.96\dfrac{\sigma}{\sqrt{n}}$.

a $1.96\sqrt{\dfrac{0.2}{30}} = .160$ **b** $1.96\sqrt{\dfrac{0.9}{30}} = .339$ **c** $1.96\sqrt{\dfrac{1.5}{30}} = .438$

8.4 Refer to Exercise 8.3. As the population variance σ^2 increases, the margin of error also increases.

8.5 The margin of error is $1.96\ SE = 1.96\dfrac{\sigma}{\sqrt{n}}$, where σ can be estimated by the sample standard deviation *s* for large values of *n*.

a $1.96\sqrt{\dfrac{4}{50}} = .554$ **b** $1.96\sqrt{\dfrac{4}{500}} = .175$ **c** $1.96\sqrt{\dfrac{4}{5000}} = .055$

8.6 Refer to Exercise 8.5. As the sample size *n* increases, the margin of error decreases.

8.13 The point estimate of μ is $\overline{x} = 39.8^{\circ}$ and the margin of error with $s = 17.2$ and $n = 50$ is

$$1.96\ SE = 1.96\frac{\sigma}{\sqrt{n}} \approx 1.96\frac{s}{\sqrt{n}} = 1.96\frac{17.2}{\sqrt{50}} = 4.768$$

8.17 **a** The point estimate for *p* is given as $\hat{p} = \dfrac{x}{n} = .51$ and the margin of error is approximately

$$1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{.51(.49)}{900}} = .0327$$

8.24 **a** $\overline{x} \pm z_{.005}\dfrac{\sigma}{\sqrt{n}} = \overline{x} \pm 2.58\dfrac{\sigma}{\sqrt{n}} \approx 34 \pm 2.58\sqrt{\dfrac{12}{38}} = 34 \pm 1.450$ or $32.550 < \mu < 35.450$.

b $\overline{x} \pm z_{.05}\dfrac{\sigma}{\sqrt{n}} = \overline{x} \pm 1.645\dfrac{\sigma}{\sqrt{n}} \approx 1049 \pm 1.645\sqrt{\dfrac{51}{65}} = 1049 \pm 1.457$ or $1047.543 < \mu < 1050.457$.

c $\overline{x} \pm z_{.025}\dfrac{\sigma}{\sqrt{n}} = \overline{x} \pm 1.96\dfrac{\sigma}{\sqrt{n}} \approx 66.3 \pm 1.96\sqrt{\dfrac{2.48}{89}} = 66.3 \pm .327$ or $65.973 < \mu < 66.627$.

8.27 The width of a 95% confidence interval for μ is given as $1.96\dfrac{\sigma}{\sqrt{n}}$. Hence,

a When $n = 100$, the width is $2\left(1.96\dfrac{10}{\sqrt{100}}\right) = 2(1.96) = 3.92$.

b When $n = 200$, the width is $2\left(1.96\dfrac{10}{\sqrt{200}}\right) = 2(1.386) = 2.772$.

c When $n = 400$, the width is $2\left(1.96\dfrac{10}{\sqrt{400}}\right) = 2(.98) = 1.96$.

8.28 Refer to Exercise 8.27.

a When the sample size is doubled, the width is decreased by $1/\sqrt{2}$.

b When the sample size is quadrupled, the width is decreased by $1/\sqrt{4} = 1/2$.

8.29 **a** A 90% confidence interval for μ is $\bar{x} \pm 1.645\frac{\sigma}{\sqrt{n}}$. Hence, its width is

$$2\left(1.645\frac{\sigma}{\sqrt{n}}\right) = 2\left(1.645\frac{10}{\sqrt{100}}\right) = 2(1.645) = 3.29$$

b A 99% confidence interval for μ is $\bar{x} \pm 2.58\frac{\sigma}{\sqrt{n}}$. Hence, its width is

$$2\left(2.58\frac{\sigma}{\sqrt{n}}\right) = 2\left(2.58\frac{10}{\sqrt{100}}\right) = 2(2.58) = 5.16$$

c Notice that as the confidence coefficient increases, so does the width of the confidence interval. If we wish to be more confident of enclosing the unknown parameter, we must make the interval wider.

8.32 **a** An approximate 95% confidence interval for p is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = .54 \pm 1.96\sqrt{\frac{.54(.46)}{400}} = .54 \pm .049$$

or $.491 < p < .589$.

b An approximate 95% confidence interval for p is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = .30 \pm 1.96\sqrt{\frac{.30(.70)}{350}} = .30 \pm .048$$

or $.252 < p < .348$.

8.34 **a** The 90% confidence interval for p is

$$\hat{p} \pm 1.645\sqrt{\frac{\hat{p}\hat{q}}{n}} = .39 \pm 1.645\sqrt{\frac{.39(.61)}{1002}} = .39 \pm .025$$

or $.365 < p < .415$.

b The 90% confidence interval for p is

$$\hat{p} \pm 1.645\sqrt{\frac{\hat{p}\hat{q}}{n}} = .53 \pm 1.645\sqrt{\frac{.53(.47)}{1002}} = .53 \pm .026$$

or $.504 < p < .556$.

8.37 **a** The 99% confidence interval for μ is

$$\bar{x} \pm 2.58\frac{s}{\sqrt{n}} = 98.25 \pm 2.58\frac{0.73}{\sqrt{130}} = 98.25 \pm .165 \text{ or } 98.085 < \mu < 98.415$$

b Since the possible values for μ given in the confidence interval does not include the value μ = 98.6, it is not likely that the true average body temperature for healthy humans is 98.6, the usual average temperature cited by physicians and others.

8.42 Similar to previous exercises. The 90% confidence interval for $\mu_1 - \mu_2$ is approximately

$$(\bar{x}_1 - \bar{x}_2) \pm 1.645\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(2.4 - 3.1) \pm 1.645\sqrt{\frac{1.44}{100} + \frac{2.64}{100}}$$

$$-0.7 \pm .332 \quad \text{or} \quad -1.032 < (\mu_1 - \mu_2) < -0.368$$

Intervals constructed in this manner will enclose $\mu_1 - \mu_2$ 90% of the time. Hence, we are fairly certain that this particular interval encloses $(\mu_1 - \mu_2)$.

8.44 Similar to previous exercises. The 95% confidence interval for $\mu_1 - \mu_2$ is approximately

$$(\bar{x}_1 - \bar{x}_2) \pm 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(21.3 - 13.4) \pm 1.96\sqrt{\frac{(2.6)^2}{30} + \frac{(1.9)^2}{30}}$$

$$7.9 \pm 1.152 \quad \text{or} \quad 6.748 < (\mu_1 - \mu_2) < 9.052$$

Intervals constructed in this manner will enclose $(\mu_1 - \mu_2)$ 95% of the time in repeated sampling. Hence, we are fairly certain that this particular interval encloses $(\mu_1 - \mu_2)$.

8.45 **a** The point estimate of the difference $\mu_1 - \mu_2$ is

$$\bar{x}_1 - \bar{x}_2 = 53,659 - 51,042 = 2617$$

and the margin of error is

$$1.96\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \approx 1.96\sqrt{\frac{2225^2}{50} + \frac{2375^2}{50}} = 902.08$$

b Since the margin of error does not allow the estimate of the difference $\mu_1 - \mu_2$ to be negative—the lower limit is $2617 - 902.08 = 1714.92$ —it is likely that the mean for chemical engineering majors is larger than the mean for computer science majors.

8.49 The 95% confidence interval for $\mu_1 - \mu_2$ is approximately

$$(\bar{x}_1 - \bar{x}_2) \pm 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(98.11 - 98.39) \pm 1.96\sqrt{\frac{.7^2}{65} + \frac{.74^2}{65}}$$

$$-.28 \pm .248 \quad \text{or} \quad -.528 < (\mu_1 - \mu_2) < -.032$$

b Since the confidence interval in part **a** has two negative endpoints, it does not contain the value $\mu_1 - \mu_2 = 0$. Hence, it is not likely that the means are equal. It appears that there is a real difference in the mean temperatures for males and females.

8.54 Calculate $\hat{p}_1 = \frac{x_1}{995} = .41$ and $\hat{p}_2 = \frac{x_2}{1094} = .44$. The approximate 95% confidence interval is

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$(.41 - .44) \pm 1.96\sqrt{\frac{.41(.59)}{995} + \frac{.44(.56)}{1094}}$$

$$-.03 \pm .042 \quad \text{or} \quad -.072 < (p_1 - p_2) < .012$$

Since the value $p_1 - p_2 = 0$ is in the confidence interval, it is possible that $p_1 = p_2$. You should not conclude that there is a difference in the proportion of Republicans and Democrats who favor mentioned the economy as an important issue in the elections.

8.56 **a** Calculate $\hat{p}_1 = \frac{410}{451} = .909$ and $\hat{p}_2 = \frac{505}{550} = .918$. The approximate 95% confidence interval is

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$(.909 - .918) \pm 1.96\sqrt{\frac{.909(.091)}{451} + \frac{.918(.082)}{550}}$$

$$-.009 \pm .035 \quad \text{or} \quad -.044 < (p_1 - p_2) < .026$$

Since the value $p_1 - p_2 = 0$ is in the confidence interval, it is possible that $p_1 = p_2$. You should not conclude that there is a difference in the proportion of fans versus non-fans who favor mandatory drug testing.

8.62 **a** The point estimate for *p* is given as $\hat{p} = \frac{x}{n} = \frac{23}{41} = .561$ and the margin of error is approximately

$$1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{.56(.44)}{41}} = .152$$

b Calculate $\hat{p}_1 = \frac{10}{32} = .3125$ and $\hat{p}_2 = \frac{23}{41} = .561$. The approximate 95% confidence interval is

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$(.3125 - .561) \pm 1.96\sqrt{\frac{.3125(.6875)}{32} + \frac{.561(.439)}{41}}$$

$$-.2485 \pm .2211 \quad \text{or} \quad -.4696 < (p_1 - p_2) < -.0274$$

8.68 It is necessary to find the sample size required to estimate a certain parameter to within a given bound with confidence $(1-\alpha)$. Recall from Section 8.5 that we may estimate a parameter with $(1-\alpha)$ confidence within the interval (estimator) $\pm z_{\alpha/2} \times$ (std error of estimator). Thus, $z_{\alpha/2} \times$ (std error of estimator) provides the margin of error with $(1-\alpha)$ confidence. The experimenter will specify a given bound B. If we let $z_{\alpha/2} \times$ (std error of estimator) $\le$ B, we will be $(1-\alpha)$ confident that the estimator will lie within B units of the parameter of interest.

For this exercise, the parameter of interest is μ, B $= 1.6$ and $1-\alpha = .95$. Hence, we must have

$$1.96\frac{\sigma}{\sqrt{n}} \le 1.6 \Rightarrow \quad 1.96\frac{12.7}{\sqrt{n}} \le 1.6$$

$$\sqrt{n} \ge \frac{1.96(12.7)}{1.6} = 15.5575$$

$$n \ge 242.04 \quad \text{or} \quad n \ge 243$$

8.69 For this exercise, B $= .04$ for the binomial estimator $\hat{p}$, where $SE(\hat{p}) = \sqrt{\frac{pq}{n}}$. Assuming maximum variation, which occurs if $p = .3$ (since we suspect that $.1 < p < .3$) and $z_{.025} = 1.96$, we have

$$1.96\sigma_{\hat{p}} \le \text{B} \Rightarrow \quad 1.96\sqrt{\frac{pq}{n}} \le \text{B}$$

$$1.96\sqrt{\frac{.3(.7)}{n}} \le .04 \Rightarrow \quad \sqrt{n} \ge \frac{1.96\sqrt{.3(.7)}}{.04} \Rightarrow n \ge 504.21 \quad \text{or} \quad n \ge 505$$

8.70 In this exercise, the parameter of interest is $\mu_1 - \mu_2$, $n_1 = n_2 = n$, and $\sigma_1^2 \approx \sigma_2^2 \approx 27.8$. Then we must have

$$z_{\alpha/2} \times (\text{std error of } \bar{x}_1 - \bar{x}_2) \le \text{B}$$

$$1.645\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le .17 \;\Rightarrow\; 1.645\sqrt{\frac{27.8}{n} + \frac{27.8}{n}} \le .17$$

$$\sqrt{n} \ge \frac{1.645\sqrt{55.6}}{.17} \;\Rightarrow\; n \ge 5206.06 \text{ or } n_1 = n_2 = 5207$$

8.74 Similar to Exercise 8.71.

$$z_{.025}\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \le .03 \;\Rightarrow\; 1.96\sqrt{\frac{(.5)(.5)}{n} + \frac{(.5)(.5)}{n}} \le .03$$

$$\sqrt{n} \ge \frac{1.96\sqrt{.5}}{.03} \;\Rightarrow\; n \ge 2134.2 \text{ or } n_1 = n_2 = 2135$$

8.80 The parameter of interest is $\mu_1 - \mu_2$, the difference in grade-point averages for the two populations of students. Assume that $n_1 = n_2 = n$, and $\sigma_1^2 \approx \sigma_2^2 \approx (.6)^2 = .36$ and that the desired bound is .2.

Then

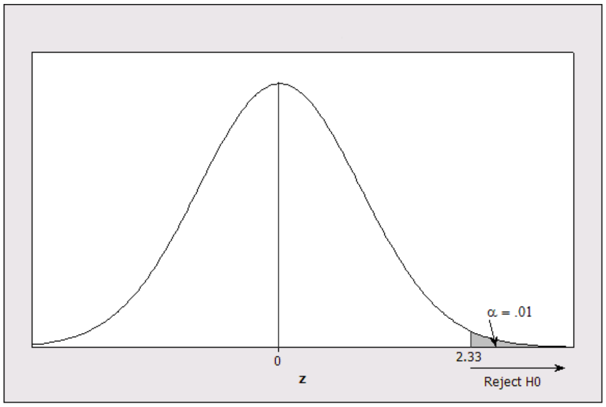
$$1.96\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le .2 \;\Rightarrow\; 1.96\sqrt{\frac{.36}{n} + \frac{.36}{n}} \le .2$$

$$\sqrt{n} \ge \frac{1.96\sqrt{.72}}{.2} \;\Rightarrow\; n \ge 69.149$$

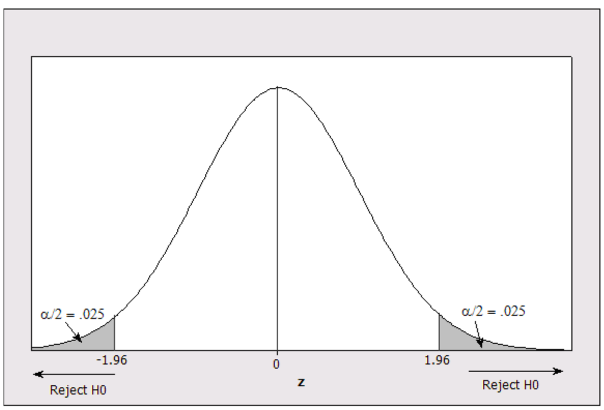
or $n_1 = n_2 = 70$ students should be included in each group.

9: Large-Sample Tests of Hypotheses

9.3 **a** The critical value that separates the rejection and nonrejection regions for a right-tailed test based on a *z*-statistic will be a value of *z* (called z_α) such that $P(z > z_\alpha) = \alpha = .01$. That is, $z_{.01} = 2.33$ (see the figure below). The null hypothesis H_0 will be rejected if $z > 2.33$.



b For a two-tailed test with $\alpha = .05$, the critical value for the rejection region cuts off $\alpha/2 = .025$ in the two tails of the *z* distribution in Figure 9.2, so that $z_{.025} = 1.96$. The null hypothesis H_0 will be rejected if $z > 1.96$ or $z < -1.96$ (which you can also write as $|z| > 1.96$).



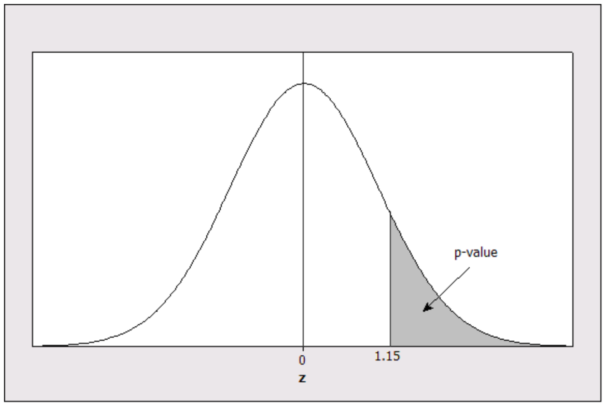
c Similar to part **a**, with the rejection region in the lower tail of the *z* distribution. The null hypothesis H_0 will be rejected if $z < -2.33$.

d Similar to part **b**, with $\alpha/2 = .005$. The null hypothesis H_0 will be rejected if $z > 2.58$ or $z < -2.58$ (which you can also write as $|z| > 2.58$).

9.4 **a** The *p*-value for a right-tailed test is the area to the right of the observed test statistic $z = 1.15$ or

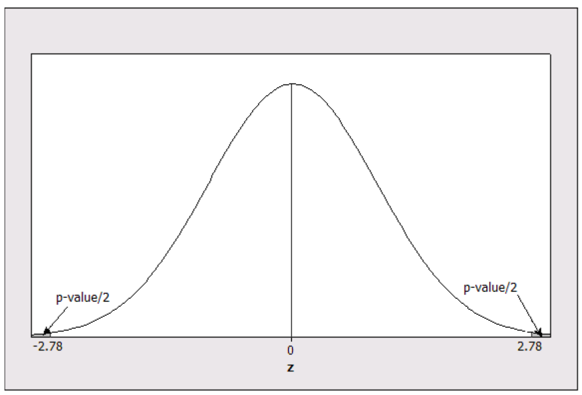
$$p\text{-value} = P(z > 1.15) = 1 - .8749 = .1251$$

This is the shaded area in the figure below.



b For a two-tailed test, the *p*-value is the probability of being as large or larger than the observed test statistic *in either tail of the sampling distribution*. As shown in the figure below, the *p*-value for $z = -2.78$ is

$$p\text{-value} = P(|z| > 2.78) = 2(.0027) = .0054$$



c The *p*-value for a left-tailed test is the area to the left of the observed test statistic $z = -1.81$ or

$$p\text{-value} = P(z < -1.81) = .0351$$

9.5 Use the guidelines for statistical significance in Section 9.3. The smaller the *p*-value, the more evidence there is in favor of rejecting H_0. For part **a**, $p\text{-value} = .1251$ is not statistically significant; H_0 is not rejected. For part **b**, $p\text{-value} = .0054$ is less than .01 and the results are highly significant; H_0 should be rejected. For part **c**, $p\text{-value} = .0351$ is between .01 and .05. The results are significant at the 5% level, but not at the 1% level (P < .05).

9.6 In this exercise, the parameter of interest is μ, the population mean. The objective of the experiment is to show that the mean exceeds 2.3.

a We want to prove the alternative hypothesis that μ is, in fact, greater then 2.3. Hence, the alternative hypothesis is

$$H_a : \mu > 2.3$$

and the null hypothesis is

$$H_0 : \mu = 2.3 .$$

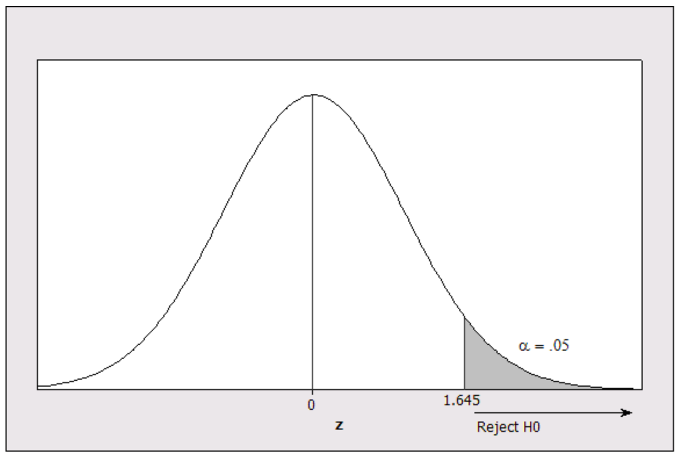
b The best estimator for μ is the sample average $\bar{x}$, and the test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

which represents the distance (measured in units of standard deviations) from $\bar{x}$ to the hypothesized mean μ. Hence, if this value is large in absolute value, one of two conclusions may be drawn. Either a very unlikely event has occurred, or the hypothesized mean is incorrect. Refer to part **a**. If $\alpha = .05$, the critical value of *z* that separates the rejection and non-rejection regions will be a value (denoted by z_0) such that

$$P(z > z_0) = \alpha = .05$$

That is, $z_0 = 1.645$ (see below). Hence, H_0 will be rejected if $z > 1.645$.



c The standard error of the mean is found using the sample standard deviation *s* to approximate the population standard deviation σ:

$$SE = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}} = \frac{.29}{\sqrt{35}} = .049$$

d To conduct the test, calculate the value of the test statistic using the information contained in the sample. Note that the value of the true standard deviation, σ, is approximated using the sample standard deviation *s*.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.4 - 2.3}{.049} = 2.04$$

The observed value of the test statistic, $z = 2.04$, falls in the rejection region and the null hypothesis is rejected. There is sufficient evidence to indicate that $\mu > 2.3$.

9.7 **a** Since this is a right-tailed test, the *p*-value is the area under the standard normal distribution to the right of $z = 2.04$:

$$p\text{-value} = P(z > 2.04) = 1 - .9793 = .0207$$

b The *p*-value, .0207, is less than $\alpha = .05$, and the null hypothesis is rejected at the 5% level of significance. There is sufficient evidence to indicate that $\mu > 2.3$.

c The conclusions reached using the **critical value approach** and the ***p*-value approach** are identical.

9.8 Refer to Exercise 9.6, in which the rejection region was given as $z > 1.645$ where

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{\bar{x} - 2.3}{.29/\sqrt{35}}$$

Solving for $\bar{x}$ we obtain the critical value of $\bar{x}$ necessary for rejection of H_0.

$$\frac{\bar{x} - 2.3}{.29/\sqrt{35}} > 1.645 \quad \Rightarrow \quad \bar{x} > 1.645\frac{.29}{\sqrt{35}} + 2.3 = 2.38$$

b-c The probability of a Type II error is defined as

$$\beta = P(\text{accept } H_0 \text{ when } H_0 \text{ is false})$$

Since the acceptance region is $\bar{x} \le 2.38$ from part **a**, β can be rewritten as

$$\beta = P(\bar{x} \le 2.38 \text{ when } H_0 \text{ is false}) = P(\bar{x} \le 2.38 \text{ when } \mu > 2.3)$$

Several alternative values of μ are given in this exercise. For $\mu = 2.4$,

$$\beta = P(\bar{x} \le 2.38 \text{ when } \mu = 2.4) = P\left(z \le \frac{2.38 - 2.4}{.29/\sqrt{35}}\right)$$

$$= P(z \le -.41) = .3409$$

For $\mu = 2.3$,

$$\beta = P(\bar{x} \le 2.38 \text{ when } \mu = 2.3) = P\left(z \le \frac{2.38 - 2.3}{.29/\sqrt{35}}\right)$$

$$= P(z \le 1.63) = .9484$$

For $\mu = 2.5$,

$$\beta = P(\bar{x} \le 2.38 \text{ when } \mu = 2.5) = P\left(z \le \frac{2.38 - 2.5}{.29/\sqrt{35}}\right)$$

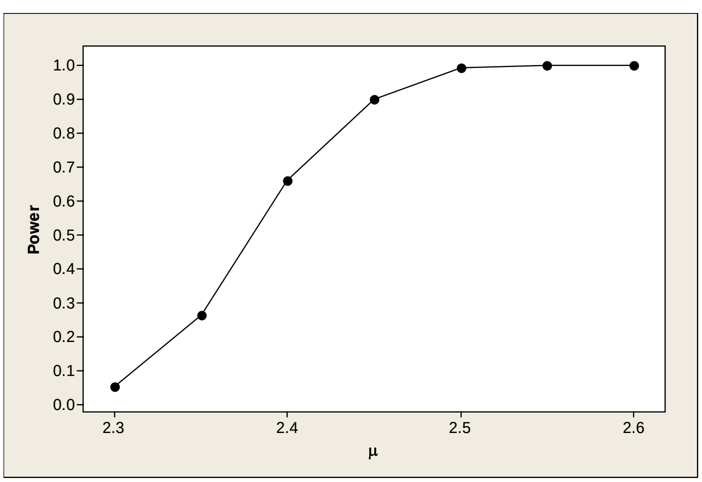
$$= P(z \le -2.45) = .0071$$

For $\mu = 2.6$,

$$\beta = P(\bar{x} \le 2.38 \text{ when } \mu = 2.6) = P\left(z \le \frac{2.38 - 2.6}{.29/\sqrt{35}}\right)$$

$$= P(z \le -4.49) \approx 0$$

d The power curve is graphed using the values calculated above and is shown below.



9.10 **a** If the airline is to determine whether or not the flight is unprofitable, they are interested in finding out whether or not $\mu < 60$ (since a flight is profitable if μ is at least 60). Hence, the alternative hypothesis is $\mathrm{H_a}: \mu < 60$ and the null hypothesis is $\mathrm{H_0}: \mu = 60$.

b Since only small values of $\bar{x}$ (and hence, negative values of z) would tend to disprove H_0 in favor of H_a, this is a one-tailed test.

c For this exercise, $n = 120$, $\bar{x} = 58$, and $s = 11$. Hence, the test statistic is

$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \approx \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{58-60}{11/\sqrt{120}} = -1.992$$

The rejection region with $\alpha = .05$ is determined by a critical value of z such that $P(z < z_0) = .05$. This value is $z_0 = -1.645$ and H_0 will be rejected if $z < -1.645$ (compare the right-tailed rejection region in Exercise 9.6). The observed value of z falls in the rejection region and H_0 is rejected. The flight is unprofitable.

9.13 **a-b** We want to test the null hypothesis that μ is, in fact, 80% against the alternative that it is not:

$$\mathrm{H_0}: \mu = 80 \quad \text{versus} \quad \mathrm{H_a}: \mu \neq 80$$

Since the exercise does not specify $\mu < 80$ or $\mu > 80$, we are interested in a two directional alternative, $\mu \neq 80$.

c The test statistic is

$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \approx \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{79.7-80}{.8/\sqrt{100}} = -3.75$$

The rejection region with $\alpha = .05$ is determined by a critical value of z such that

$$P(z < -z_0) + P(z > z_0) = \frac{\alpha}{2} + \frac{\alpha}{2} = .05$$

This value is $z_0 = 1.96$ (see the figure in Exercise 9.3**b**). Hence, H_0 will be rejected if $z > 1.96$ or $z < -1.96$. The observed value, $z = -3.75$, falls in the rejection region and H_0 is rejected. There is sufficient evidence to refute the manufacturer's claim. The probability that we have made an incorrect decision is $\alpha = .05$.

9.15 **a** The hypothesis to be tested is

$$\mathrm{H_0}: \mu = 7.4 \quad \text{versus} \quad \mathrm{H_a}: \mu > 7.4$$

and the test statistic is

$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \approx \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{7.9-7.4}{1.9/\sqrt{100}} = 2.63$$

with p-value $= P(z > 2.63) = 1 - .9957 = .0043$. To draw a conclusion from the p-value, use the guidelines for statistical significance in Section 9.3. Since the p-value is less than .01, the test results are highly significant. We can reject H_0 at both the 1% and 5% levels of significance.

b You could claim that you work significantly fewer hours than those without a college education.

c If you were not a college graduate, you might just report that you work an average of more than 7.4 hours per week..

9.16 **a** The hypothesis to be tested is

$$\mathrm{H_0}: \mu = 98.6 \quad \text{versus} \quad \mathrm{H_a}: \mu \neq 98.6$$

and the test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \approx \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{98.25 - 98.6}{.73/\sqrt{130}} = -5.47$$

with p-value $= P(z < -5.47) + P(z > 5.47) \approx 2(0) = 0$. Alternatively, we could write $p\text{-value} = 2P(z < -5.47) < 2(.0002) = .0004$ With $\alpha = .05$, the p-value is less than α and H_0 is rejected. There is sufficient evidence to indicate that the average body temperature for healthy humans is different from 98.6.

b-c Using the critical value approach, we set the null and alternative hypotheses and calculate the test statistic as in part **a**. The rejection region with $\alpha = .05$ is $|z| > 1.96$. The observed value, $z = -5.47$, does fall in the rejection region and H_0 is rejected. The conclusion is the same is in part **a**.

d How did the doctor record 1 million temperatures in 1868? The technology available at that time makes this a difficult if not impossible task. It may also have been that the instruments used for this research were not entirely accurate.

9.18 **a-b** The hypothesis of interest is one-tailed:

$$H_0: \mu_1 - \mu_2 = 0 \quad \text{versus} \quad H_a: \mu_1 - \mu_2 > 0$$

c The test statistic, calculated under the assumption that $\mu_1 - \mu_2 = 0$, is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

with σ_1^2 and σ_2^2 known, or estimated by s_1^2 and s_2^2, respectively. For this exercise,

$$z \approx \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{11.6 - 9.7}{\sqrt{\frac{27.9}{80} + \frac{38.4}{80}}} = 2.09$$

a value which lies slightly more than two standard deviations from the hypothesized difference of zero. This would be a somewhat unlikely observation, if H_0 is true.

d The p-value for this one-tailed test is

$$p\text{-value} = P(z > 2.09) = 1 - .9817 = .0183$$

Since the p-value is not less than $\alpha = .01$, the null hypothesis cannot be rejected at the 1% level. There is insufficient evidence to conclude that $\mu_1 - \mu_2 > 0$.

e Using the critical value approach, the rejection region, with $\alpha = .01$, is $z > 2.33$ (see Exercise 9.3**a**). Since the observed value of z does not fall in the rejection region, H_0 is not rejected. There is insufficient evidence to indicate that $\mu_1 - \mu_2 > 0$, or $\mu_1 > \mu_2$.

9.20 The probability that you are making an incorrect decision is influenced by the fact that if $\mu_1 - \mu_2 = 0$, it is just as likely that $\bar{x}_1 - \bar{x}_2$ will be positive as that it will be negative. Hence, a two-tailed rejection region *must* be used. Choosing a one-tailed region after determining the sign of $\bar{x}_1 - \bar{x}_2$ simply tells us which of the two pieces of the rejection region is being used. Hence,

$$\alpha = P(\text{reject } H_0 \text{ when } H_0 \text{ true}) = P(z > 1.645 \text{ or } z < -1.645 \text{ when } H_0 \text{ true})$$
$$= \alpha_1 + \alpha_2 = .05 + .05 = .10$$

which is twice what the experimenter thinks it is. Hence, one cannot choose the rejection region after the test is performed.

9.22 **a** The hypothesis of interest is one-tailed:

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{versus} \quad H_a : \mu_1 - \mu_2 > 0$$

The test statistic, calculated under the assumption that $\mu_1 - \mu_2 = 0$, is

$$z \approx \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{73 - 63}{\sqrt{\frac{(25)^2}{400} + \frac{(28)^2}{400}}} = 5.33$$

The rejection region with $\alpha = .01$, is $z > 2.33$ and H_0 is rejected. There is evidence to indicate that $\mu_1 - \mu_2 > 0$, or $\mu_1 > \mu_2$. The average per-capita beef consumption has decreased in the last ten years. (Alternatively, the *p*-value for this test is the area to the right of $z = 5.33$ which is very close to zero and less than $\alpha = .01$.)

b For the difference $\mu_1 - \mu_2$ in the population means this year and ten years ago, the 99% lower confidence bound uses $z_{.01} = 2.33$ and is calculated as

$$(\bar{x}_1 - \bar{x}_2) - 2.33\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (73 - 63) - 2.33\sqrt{\frac{25^2}{400} + \frac{28^2}{400}}$$

$$10 - 4.37 \quad \text{or} \quad (\mu_1 - \mu_2) > 5.63$$

Since the difference in the means is positive, you can again conclude that there has been a decrease in average per-capita beef consumption over the last ten years. In addition, it is likely that the average consumption has decreased by more than 5.63 pounds per year.

9.24 The hypothesis of interest is two-tailed:

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{versus} \quad H_a : \mu_1 - \mu_2 \neq 0$$

and the test statistic, calculated under the assumption that $\mu_1 - \mu_2 = 0$, is

$$z \approx \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{53,659 - 51,042}{\sqrt{\frac{2225^2}{50} + \frac{2375^2}{50}}} = 5.69$$

The rejection region, with $\alpha = .05$, is $|z| > 1.96$ and H_0 is rejected. There is evidence to indicate a difference in the means for the graduates in chemical engineering and computer science.

b The conclusions are the same.

9.29 **a** The hypothesis of interest is two-tailed:

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{versus} \quad H_a : \mu_1 - \mu_2 \neq 0$$

and the test statistic is

$$z \approx \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{98.11 - 98.39}{\sqrt{\frac{.7^2}{65} + \frac{.74^2}{65}}} = -2.22$$

with $p\text{-value} = P(|z| > 2.22) = 2(1 - .9868) = .0264$. Since the *p*-value is between .01 and .05, the null hypothesis is rejected, and the results are significant. There is evidence to indicate a difference in the mean temperatures for men versus women.

b Since the *p*-value = .0264, we can reject H_0 at the 5% level (*p*-value < .05), but not at the 1% level (*p*-value > .01). Using the guidelines for significance given in Section 9.3 of the text, we declare the results statistically *significant*, but not *highly significant.*

9.30 **a** The hypothesis of interest concerns the binomial parameter *p* and is one-tailed:

$$H_0 : p = .3 \quad \text{versus} \quad H_a : p < .3$$

b The rejection region is one-tailed, with $\alpha = .05$, or $z < -1.645$.

c It is given that $x = 279$ and $n = 1000$, so that $\hat{p} = \frac{x}{n} = \frac{279}{1000} = .279$. The test statistic is then

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.279 - .3}{\sqrt{\frac{.3(.7)}{1000}}} = -1.449$$

Since the observed value does not fall in the rejection region, H_0 is not rejected. We cannot conclude that $p < .3$.

9.33 **a** The hypothesis to be tested involves the binomial parameter *p*:

$$H_0 : p = .15 \quad \text{versus} \quad H_a : p < .15$$

where *p* is the proportion of parents who describe their children as overweight. For this test, $x = 68$ and $n = 750$, so that $\hat{p} = \frac{x}{n} = \frac{68}{750} = .091$, the test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.091 - .15}{\sqrt{\frac{.15(.85)}{750}}} = -4.53$$

b The rejection region is one-tailed, with $z < -1.645$ with $\alpha = .05$. Since the test statistic falls in the rejection region, the null hypothesis is rejected. There is sufficient evidence to indicate that the proportion of parents who describe their children as overweight is less than the actual proportion reported by the American Obesity Association.

c The *p*-value is calculated as

$p\text{-value} = P(z < -4.53) < .0002$ or $p\text{-value} \approx 0$. Since the *p*-value is less than .05, the null hypothesis is rejected as in part **b**.

9.35 **a-b** Since the survival rate without screening is $p = 2/3$, the survival rate with an effective program may be greater than 2/3. Hence, the hypothesis to be tested is

$$H_0 : p = 2/3 \quad \text{versus} \quad H_a : p > 2/3$$

c With $\hat{p} = \frac{x}{n} = \frac{164}{200} = .82$, the test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.82 - 2/3}{\sqrt{\frac{(2/3)(1/3)}{200}}} = 4.6$$

The rejection region is one-tailed, with $\alpha = .05$ or $z > 1.645$ and H_0 is rejected. The screening program seems to increase the survival rate.

d For the one-tailed test,

$$p\text{-value} = P(z > 4.6) < 1 - .9998 = .0002$$

That is, H_0 can be rejected for any value of $\alpha \geq .0002$. The results are *highly significant.*

9.40 The hypothesis of interest is

$$H_0 : p = .35 \quad \text{versus} \quad H_a : p \neq .35$$

with $\hat{p} = \frac{x}{n} = \frac{123}{300} = .41$, the test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.41 - .35}{\sqrt{\frac{.35(.65)}{300}}} = 2.17$$

The rejection region with $\alpha = .01$ is $|z| > 2.58$ and the null hypothesis is not rejected. (Alternatively, we could calculate $p\text{-value} = 2P(z < -2.17) = 2(.0150) = .0300$. Since this *p*-value is greater than .01, the null hypothesis is not rejected.) There is insufficient evidence to indicate that the percentage of adults who say that they always vote is different from the percentage reported in *Time*.

9.42 **a** Since it is necessary to detect either $p_1 > p_2$ or $p_1 < p_2$, a two-tailed test is necessary:

$$H_0 : p_1 - p_2 = 0 \quad \text{versus} \quad H_a : p_1 - p_2 \neq 0$$

b The standard error of $\hat{p}_1 - \hat{p}_2$ is

$$\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

In order to evaluate the standard error, estimates for p_1 and p_2 must be obtained, using the assumption that $p_1 - p_2 = 0$. Because we are assuming that $p_1 - p_2 = 0$, the best estimate for this common value will be

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{74 + 81}{140 + 140} = .554$$

and the estimated standard error is

$$\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{.554(.446)\left(\frac{2}{140}\right)} = .0594$$

c Calculate $\hat{p}_1 = \frac{74}{140} = .529$ and $\hat{p}_2 = \frac{81}{140} = .579$. The test statistic, based on the sample data will be

$$z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} \approx \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.529 - .579}{.0594} = -.84$$

This is a likely observation if H_0 is true, since it lies less than one standard deviation below $p_1 - p_2 = 0$.

d Calculate the two tailed $p\text{-value} = P(|z| > .84) = 2(.2005) = .4010$. Since this *p*-value is greater than .01, H_0 is not rejected. There is no evidence of a difference in the two population proportions.

e The rejection region with $\alpha = .01$, or $|z| > 2.58$ and H_0 is not rejected. There is no evidence of a difference in the two population proportions.

9.45 **a** The hypothesis of interest is:

$$H_0 : p_1 - p_2 = 0 \quad \text{versus} \quad H_a : p_1 - p_2 < 0$$

Calculate $\hat{p}_1 = .36$, $\hat{p}_2 = .60$ and $\hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{18 + 30}{50 + 50} = .48$. The test statistic is then

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.36 - .60}{\sqrt{.48(.52)(1/50 + 1/50)}} = -2.40$$

The rejection region, with $\alpha = .05$, is $z < -1.645$ and H_0 is rejected. There is evidence of a difference in the proportion of survivors for the two groups.

b From Section 8.7, the approximate 95% confidence interval is

$$(\hat{p}_1-\hat{p}_2)\pm 1.96\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$$
$$(.36-.60)\pm 1.96\sqrt{\frac{.36(.64)}{50}+\frac{.60(.40)}{50}}$$
$$-.24\pm .19 \quad \text{or} \quad -.43<(p_1-p_2)<-.05$$

9.48 **a** The hypothesis of interest is
$$H_0: p_1-p_2=0 \quad \text{versus} \quad H_a: p_1-p_2>0$$

Calculate $\hat{p}_1=\frac{40}{2266}=.018$, $\hat{p}_2=\frac{21}{2266}=.009$, and $\hat{p}=\frac{x_1+x_2}{n_1+n_2}=\frac{40+21}{4532}=.013$. The test statistic is then
$$z=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}=\frac{.018-.009}{\sqrt{.013(.987)(1/2266+1/2266)}}=2.67$$
The rejection region, with $\alpha=.01$, is $z>2.33$ and H_0 is rejected. There is sufficient evidence to indicate that the risk of dementia is higher for patients using *Prempro.*

9.50 **a** Since the two treatments were randomly assigned, the randomization procedure can be implemented as each patient becomes available for treatment. Choose a random number between 0 and 9 for each patient. If the patient receives a number between 0 and 4, the assigned drug is *aspirin*. If the patient receives a number between 5 and 9, the assigned drug is *clopidogrel*.
b Assume that $n_1=7720$ and $n_2=7780$. It is given that $\hat{p}_1=.054$, $\hat{p}_2=.038$, so that
$$\hat{p}=\frac{n_1\hat{p}_1+n_2\hat{p}_2}{n_1+n_2}=\frac{7720(.054)+7780(.038)}{15,500}=.046.$$
The test statistic is then
$$z=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}=\frac{.054-.038}{\sqrt{.046(.954)(1/7720+1/7780)}}=4.75$$
with p-value $=P(|z|>4.75)<2(.0002)=.0004$. Since the p-value is less than .01, the results are statistically significant. There is sufficient evidence to indicate a difference in the proportions for the two treatment groups.
c Clopidogrel would be the preferred treatment, as long as there are no dangerous side effects.

9.75 The hypothesis to be tested is
$$H_0: \mu=5 \quad \text{versus} \quad H_a: \mu>5$$
and the test statistic is
$$z=\frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}\approx\frac{\overline{x}-\mu_0}{s/\sqrt{n}}=\frac{7.2-5}{6.2/\sqrt{38}}=2.19$$

The rejection region with $\alpha=.01$ is $z>2.33$. Since the observed value, $z=2.19$, does not fall in the rejection region and H_0 is not rejected. The data do not provide sufficient evidence to indicate that the mean ppm of PCBs in the population of game birds exceeds the FDA's recommended limit of 5 ppm.

9.76 Refer to Exercise 9.75, in which the rejection region was given as $z>2.33$ where

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{\bar{x} - 2.3}{.29/\sqrt{35}}$$

Solving for $\bar{x}$ we obtain the critical value of $\bar{x}$ necessary for rejection of H_0.

$$\frac{\bar{x} - 5}{6.2/\sqrt{38}} > 2.33 \quad \Rightarrow \quad \bar{x} > 2.33\frac{6.2}{\sqrt{38}} + 5 = 7.34$$

The probability of a Type II error is defined as

$$\beta = P(\text{accept } H_0 \text{ when } H_0 \text{ is false})$$

Since the acceptance region is $\bar{x} \le 7.34$ from part **a**, β can be rewritten as

$$\beta = P(\bar{x} \le 7.34 \text{ when } H_0 \text{ is false}) = P(\bar{x} \le 7.34 \text{ when } \mu > 5)$$

Several alternative values of μ are given in this exercise.

a For $\mu = 6$,

$$\beta = P(\bar{x} \le 7.34 \text{ when } \mu = 6) = P\left(z \le \frac{7.34 - 6}{6.2/\sqrt{38}}\right)$$

$$= P(z \le 1.33) = .9082$$

and $1 - \beta = 1 - .9082 = .0918$.

b For $\mu = 7$,

$$\beta = P(\bar{x} \le 7.34 \text{ when } \mu = 7) = P\left(z \le \frac{7.34 - 7}{6.2/\sqrt{38}}\right)$$

$$= P(z \le .34) = .6331$$

and $1 - \beta = 1 - .6331 = .3669$.

c For $\mu = 8$,

$$1 - \beta = 1 - P(\bar{x} \le 7.34 \text{ when } \mu = 8)$$

$$= 1 - P\left(z \le \frac{7.34 - 8}{6.2/\sqrt{38}}\right)$$

$$= 1 - P(z \le -.66) = .7454$$

For $\mu = 9$,

$$1 - \beta = 1 - P(\bar{x} \le 7.34 \text{ when } \mu = 9)$$

$$= 1 - P\left(z \le \frac{7.34 - 9}{6.2/\sqrt{38}}\right)$$

$$= 1 - P(z \le -1.65) = .9505$$

For $\mu = 10$,

$$1 - \beta = 1 - P(\bar{x} \le 7.34 \text{ when } \mu = 10)$$

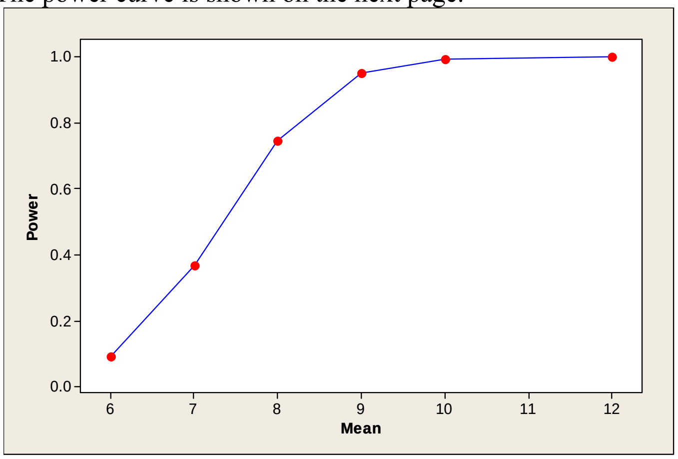
$$= 1 - P\left(z \le \frac{7.34 - 10}{6.2/\sqrt{38}}\right)$$

$$= 1 - P(z \le -2.64) = .9959$$

For $\mu = 12$,

$$1-\beta = 1-P(\bar{x} \le 7.34 \text{ when } \mu = 12)$$
$$= 1-P\left(z \le \frac{7.34-12}{6.2/\sqrt{38}}\right)$$
$$= 1-P(z \le -4.63) \approx 1$$

d The power curve is shown on the next page.



You can see that the power becomes greater than or equal to .90 for a value of μ a little smaller than $\mu = 9$. To find the exact value, we need to solve for μ in the equation:

$$1-\beta = 1-P(\bar{x} \le 7.34) = 1-P\left(z \le \frac{7.34-\mu}{6.2/\sqrt{38}}\right) = .90$$

$$\text{or } P\left(z \le \frac{7.34-\mu}{6.2/\sqrt{38}}\right) = .10$$

From Table 3, the value of *z* that cuts off .10 in the lower tail of the *z*-distribution is $z = -1.28$, so that

$$\frac{7.34-\mu}{6.2/\sqrt{38}} = -1.28$$
$$\mu = 7.34 + 1.28\frac{6.2}{\sqrt{38}} = 8.63.$$

10: Inference from Small Samples

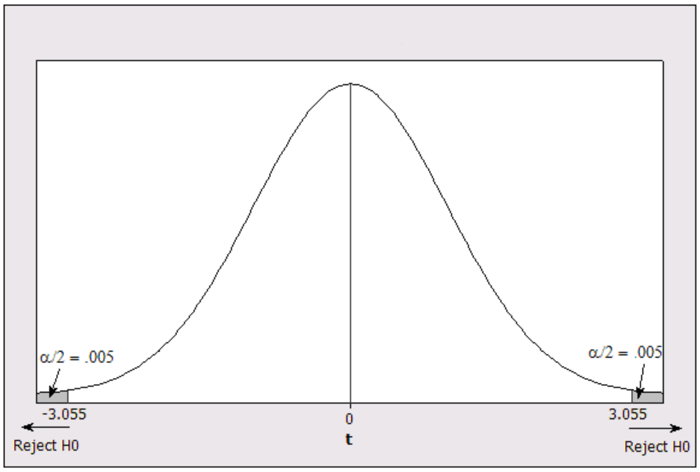
10.1 Refer to Table 4, Appendix I, indexing *df* along the left or right margin and t_α across the top.

a $t_{.05} = 2.015$ with 5 *df* b $t_{.025} = 2.306$ with 8 *df*

c $t_{.10} = 1.330$ with 18 *df* c $t_{.025} \approx 1.96$ with 30 *df*

10.2 The value $P(t > t_a) = a$ is the tabled entry for a particular number of degrees of freedom.

a For a two-tailed test with $\alpha = .01$, the critical value for the rejection region cuts off $\alpha/2 = .005$ in the two tails of the *t* distribution shown below, so that $t_{.005} = 3.055$. The null hypothesis H_0 will be rejected if $t > 3.055$ or $t < -3.055$ (which you can also write as $|t| > 3.055$).



b For a right-tailed test, the critical value that separates the rejection and nonrejection regions for a right tailed test based on a *t*-statistic will be a value of *t* (called t_α) such that $P(t > t_\alpha) = \alpha = .05$ and $df = 16$. That is, $t_{.05} = 1.746$. The null hypothesis H_0 will be rejected if $t > 1.746$.

c For a two-tailed test with $\alpha/2 = .025$ and $df = 25$, H_0 will be rejected if $|t| > 2.060$.

d For a left-tailed test with $\alpha = .01$ and $df = 7$, H_0 will be rejected if $t < -2.998$.

10.3 **a** The *p*-value for a two-tailed test is defined as

$$p\text{-value} = P(|t| > 2.43) = 2P(t > 2.43)$$

so that

$$P(t > 2.43) = \frac{1}{2}\, p\text{-value}$$

Refer to Table 4, Appendix I, with $df = 12$. The exact probability, $P(t > 2.43)$ is unavailable; however, it is evident that $t = 2.43$ falls between $t_{.025} = 2.179$ and $t_{.01} = 2.681$. Therefore, the area to the right of $t = 2.43$ must be between .01 and .025. Since

$$.01 < \frac{1}{2}\, p\text{-value} < .025$$

the *p*-value can be approximated as

$$.02 < p\text{-value} < .05$$

b For a right-tailed test, $p\text{-value} = P(t > 3.21)$ with $df = 16$. Since the value $t = 3.21$ is larger than $t_{.005} = 2.921$, the area to its right must be less than .005 and you can bound the *p*-value as

$$p\text{-value} < .005$$

c For a two-tailed test, $p\text{-value} = P(|t| > 1.19) = 2P(t > 1.19)$, so that $P(t > 1.19) = \frac{1}{2}p\text{-value}$. From Table 4 with $df = 25$, $t = 1.19$ is smaller than $t_{.10} = 1.316$ so that

$$\frac{1}{2}p\text{-value} > .10 \quad \text{and} \quad p\text{-value} > .20$$

d For a left-tailed test, $p\text{-value} = P(t < -8.77) = P(t > 8.77)$ with $df = 7$. Since the value $t = 8.77$ is larger than $t_{.005} = 3.499$, the area to its right must be less than .005 and you can bound the *p*-value as

$$p\text{-value} < .005$$

10.9 **a** Similar to previous exercises. The hypothesis to be tested is

$$H_0 : \mu = 100 \quad \text{versus} \quad H_a : \mu < 100$$

Calculate $\bar{x} = \frac{\sum x_i}{n} = \frac{1797.095}{20} = 89.85475$

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \frac{165{,}697.7081 - \frac{(1797.095)^2}{20}}{19} = 222.1150605 \quad \text{and} \quad s = 14.9035$$

The test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{89.85475 - 100}{\frac{14.9035}{\sqrt{20}}} = -3.044$$

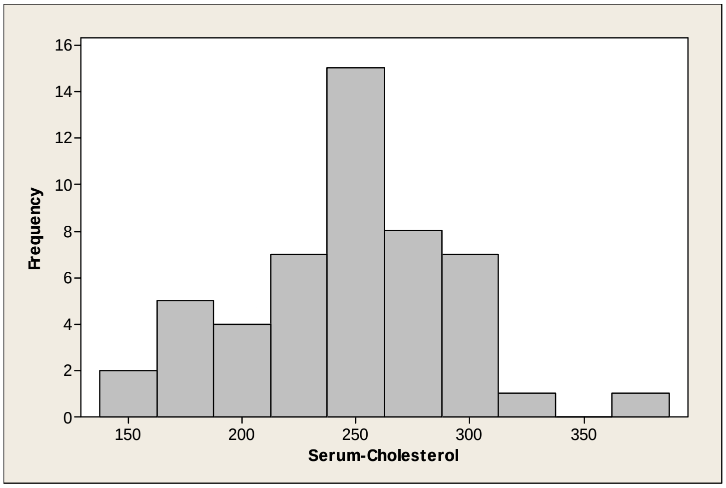
The critical value of *t* with $\alpha = .01$ and $n-1 = 19$ degrees of freedom is $t_{.01} = 2.539$ and the rejection region is $t < -2.539$. The null hypothesis is rejected and we conclude that μ is less than 100 DL.

b The 95% upper one-sided confidence bound, based on $n-1 = 19$ degrees of freedom, is

$$\bar{x} + t_{.05}\frac{s}{\sqrt{n}} \Rightarrow 89.85475 + 2.539\frac{14.90352511}{\sqrt{20}} \Rightarrow \mu < 98.316$$

This confirms the results of part **a** in which we concluded that the mean is less than 100 DL.

10.16 **a** Answers will vary. A typical histogram generated by ***Minitab*** shows that the data are approximately mound-shaped.



b Calculate $\bar{x} = \frac{\sum x_i}{n} = \frac{12348}{50} = 246.96$

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \frac{3{,}156{,}896 - \frac{(12348)^2}{50}}{49} = 2192.52898 \quad \text{and} \quad s = 46.8244$$

Table 4 does not give a value of *t* with area .025 to its right. If we are conservative, and use the value of *t* with *df* = 29, the value of *t* will be $t_{.025} = 2.045$, and the approximate 95% confidence interval is

$$\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}} \Rightarrow 246.96 \pm 2.045\frac{46.8244}{\sqrt{50}} \Rightarrow 246.96 \pm 13.54$$

or $233.42 < \mu < 260.50$.

10.17 Refer to Exercise 10.16. If we use the large sample method of Chapter 8, the large sample confidence interval is

$$\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} \Rightarrow 246.96 \pm 1.96\frac{46.8244}{\sqrt{50}} \Rightarrow 246.96 \pm 12.98$$

or $233.98 < \mu < 259.94$. The intervals are fairly similar, which is why we choose to approximate the sampling distribution of $\frac{\bar{x}-\mu}{s/\sqrt{n}}$ with a *z* distribution when $n > 30$.

10.24 **a** If the antiplaque rinse is effective, the plaque buildup should be less for the group using the antiplaque rinse. Hence, the hypothesis to be tested is

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{versus} \quad H_a : \mu_1 - \mu_2 > 0$$

b The pooled estimator of σ^2 is calculated as

$$s^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{6(.32)^2 + 6(.32)^2}{7+7-2} = .1024$$

and the test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{1.26 - .78}{\sqrt{.1024\left(\frac{1}{7}+\frac{1}{7}\right)}} = 2.806$$

The rejection region is one-tailed, based on $n_1 + n_2 - 2 = 12$ degrees of freedom. With $\alpha = .05$, from Table 4, the rejection region is $t > t_{.05} = 1.782$ and H_0 is rejected. There is evidence to indicate that the rinse is effective.

c The p-value is

$$p\text{-value} = P(t > 2.806)$$

From Table 4 with $df = 12$, $t = 2.806$ is between two tabled entries $t_{.005} = 3.055$ and $t_{.01} = 2.681$, we can conclude that

$$.005 < p\text{-value} < .01$$

10.27 **a** Check the ratio of the two variances using the rule of thumb given in this section:

$$\frac{\text{larger } s^2}{\text{smaller } s^2} = \frac{2.78095}{.17143} = 16.22$$

which is greater than three. Therefore, it is not reasonable to assume that the two population variances are equal.

b You should use the unpooled t test with Satterthwaite's approximation to the degrees of freedom for testing

$$\text{H}_0 : \mu_1 - \mu_2 = 0 \quad \text{versus} \quad \text{H}_a : \mu_1 - \mu_2 \neq 0$$

The test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{3.73 - 4.8}{\sqrt{\frac{2.78095}{15} + \frac{.17143}{15}}} = -2.412$$

with

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{(.185397 + .0114287)^2}{.002455137 + .00000933} = 15.7$$

With $df \approx 15$, the p-value for this test is bounded between .02 and .05 so that H_0 can be rejected at the 5% level of significance. There is evidence of a difference in the mean number of uncontaminated eggplants for the two disinfectants.

10.28 **a** Use your scientific calculator or the computing formulas to find:

$$\bar{x}_1 = .0125 \qquad s_1^2 = .000002278 \qquad s_1 = .001509$$

$$\bar{x}_2 = .0138 \qquad s_2^2 = .000003733 \qquad s_2 = .001932$$

Since the ratio of the variances is less than 3, you can use the pooled t test, calculating

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{9(.000002278) + 9(.000003733)}{18} = .000003006$$

and the test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.0125 - .0138}{\sqrt{s^2\left(\frac{1}{10} + \frac{1}{10}\right)}} = -1.68$$

For a two-tailed test with $df = 18$, the p-value can be bounded using Table 4 so that

$$.05 < \frac{1}{2} p\text{-value} < .10 \quad \text{or} \quad .10 < p\text{-value} < .20$$

Since the p-value is greater than .10, $\text{H}_0 : \mu_1 - \mu_2 = 0$ is not rejected. There is insufficient evidence to indicate that there is a difference in the mean titanium contents for the two methods.

b A 95% confidence interval for $(\mu_1 - \mu_2)$ is given as

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm t_{.025}\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$(.0125 - .0138) \pm 2.101\sqrt{s^2\left(\frac{1}{10} + \frac{1}{10}\right)}$$

$$-.0013 \pm .0016 \quad \text{or} \quad -.0029 < \left(\mu_1 - \mu_2\right) < .0003$$

Since $\mu_1 - \mu_2 = 0$ falls in the confidence interval, the conclusion of part **a** is confirmed. *This particular data set is very susceptible to rounding error. You need to carry as much accuracy as possible to obtain accurate results.*

10.29 **a** The ***Minitab*** stem and leaf plots are shown below. Notice the mounded shapes which justify the assumption of normality.

Stem-and-Leaf Display: Generic, Sunmaid

```
Stem-and-leaf of Generic  N  = 14          Stem-and-leaf of Sunmaid  N  = 14
Leaf Unit = 0.10                           Leaf Unit = 0.10

 1   24  0                                 1  22  0
 4   25  000                               1  23
(5)  26  00000                             5  24  0000
 5   27  00                                7  25  00
 3   28  000                               7  26
                                            7  27  0
                                            6  28  0000
                                            2  29  0
                                            1  30  0
```

b Use your scientific calculator or the computing formulas to find:

$$\bar{x}_1 = 26.214 \qquad s_1^2 = 1.565934 \qquad s_1 = 1.251$$

$$\bar{x}_2 = 26.143 \qquad s_2^2 = 5.824176 \qquad s_2 = 2.413$$

Since the ratio of the variances is greater than 3, you must use the unpooled *t* test with Satterthwaite's approximate *df*.

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \approx 19$$

c For testing $H_0: \mu_1 - \mu_2 = 0$ versus $H_a: \mu_1 - \mu_2 \neq 0$, the test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{26.214 - 26.143}{\sqrt{\frac{1.565934}{14} + \frac{5.824176}{14}}} = .10$$

For a two-tailed test with $df = 19$, the *p*-value can be bounded using Table 4 so that

$$\frac{1}{2} p\text{-value} > .10 \quad \text{or} \quad p\text{-value} > .20$$

Since the *p*-value is greater than .10, $H_0: \mu_1 - \mu_2 = 0$ is not rejected. There is insufficient evidence to indicate that there is a difference in the mean number of raisins per box.

10.31 **a** If swimmer 2 is faster, his(her) average time should be less than the average time for swimmer 1. Therefore, the hypothesis of interest is

$$H_0: \mu_1 - \mu_2 = 0 \quad \text{versus} \quad H_a: \mu_1 - \mu_2 > 0$$

and the preliminary calculations are as follows:

Swimmer 1	Swimmer 2
$\sum x_{1i} = 596.46$	$\sum x_{2i} = 596.27$
$\sum x_{1i}^2 = 35576.6976$	$\sum x_{2i}^2 = 35554.1093$
$n_1 = 10$	$n_2 = 10$

Then

$$s^2 = \frac{\sum x_{1i}^2 - \frac{\left(\sum x_{1i}\right)^2}{n_1} + \sum x_{2i}^2 - \frac{\left(\sum x_{2i}\right)^2}{n_2}}{n_1 + n_2 - 2}$$

$$= \frac{35576.6976 - \frac{(596.46)^2}{10} + 35554.1093 - \frac{(596.27)^2}{10}}{5+5-2} = .03124722$$

Also, $\bar{x}_1 = \frac{596.46}{10} = 59.646$ and $\bar{x}_2 = \frac{596.27}{10} = 59.627$

The test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{59.646 - 59.627}{\sqrt{.03124722\left(\frac{1}{10} + \frac{1}{10}\right)}} = 0.24$$

For a one-tailed test with $df = n_1 + n_2 - 2 = 18$, the p-value can be bounded using Table 4 so that $p\text{-value} > .10$, and H_0 is not rejected. There is insufficient evidence to indicate that swimmer 2's average time is still faster than the average time for swimmer 1.

10.38 **a** A paired-difference test is used, since the two samples are not independent (for any given city, Allstate and 21st Century premiums will be related).

b The hypothesis of interest is

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{or} \quad H_0 : \mu_d = 0$$
$$H_a : \mu_1 - \mu_2 \neq 0 \quad \text{or} \quad H_a : \mu_d \neq 0$$

where μ_1 is the average for Allstate insurance and μ_2 is the average cost for 21st Century insurance. The table of differences, along with the calculation of $\bar{d}$ and s_d, is presented below.

City	1	2	3	4	Totals
d_i	389	207	222	357	1175
d_i^2	151,321	42,849	49,284	127,449	370,903

$$\bar{d} = \frac{\sum d_i}{n} = \frac{1175}{4} = 293.75 \quad \text{and}$$

$$s_d = \sqrt{\frac{\sum d_i^2 - \frac{\left(\sum d_i\right)^2}{n}}{n-1}} = \sqrt{\frac{370{,}903 - \frac{(1175)^2}{4}}{3}} = \sqrt{8582.25} = 92.64043$$

The test statistic is

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{293.75 - 0}{\frac{92.64043}{\sqrt{4}}} = 6.342$$

with $n - 1 = 3$ degrees of freedom. The rejection region with $\alpha = .01$ is $|t| > t_{.005} = 5.841$, and H_0 is rejected. There is sufficient evidence to indicate a difference in the average premiums for Allstate and 21st Century.

c $p\text{-value} = P(|t| > 6.342) = 2P(t > 6.342)$. Since $t = 6.342$ is greater than $t_{.005} = 5.841$,

$$p\text{-value} < 2(.005) \Rightarrow p\text{-value} < .01 .$$

d A 99% confidence interval for $\mu_1 - \mu_2 = \mu_d$ is

$$\bar{d} \pm t_{.005}\frac{s_d}{\sqrt{n}} \;\Rightarrow\; 293.75 \pm 5.841\frac{92.64043}{\sqrt{4}} \;\Rightarrow\; 293.75 \pm 270.556$$

or $23.194 < (\mu_1 - \mu_2) < 564.306$.

e The four cities in the study were not necessarily a random sample of cities from throughout the United States. Therefore, you cannot make valid comparisons between Allstate and 21st Century for the United States in general.

10.43 **a** A paired-difference test is used, since the two samples are not random and independent (at any location, the ground and air temperatures are related). The hypothesis of interest is

$$H_0 : \mu_1 - \mu_2 = 0 \quad H_a : \mu_1 - \mu_2 \neq 0$$

The table of differences, along with the calculation of $\bar{d}$ and s_d^2, is presented below.

Location	1	2	3	4	5	Total
d_i	–.4	–2.7	–1.6	–1.7	–1.5	–7.9

$$\bar{d} = \frac{\sum d_i}{n} = \frac{-7.9}{5} = -1.58$$

$$s_d^2 = \frac{\sum d_i^2 - \frac{(\sum d_i)^2}{n}}{n-1} = \frac{15.15 - \frac{(-7.9)^2}{5}}{4} = .667 \quad \text{and} \quad s_d = .8167$$

and the test statistic is

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{-1.58 - 0}{\frac{.8167}{\sqrt{5}}} = -4.326$$

A rejection region with $\alpha = .05$ and $df = n - 1 = 4$ is $|t| > t_{.025} = 2.776$, and H_0 is rejected at the 5% level of significance. We conclude that the air-based temperature readings are biased.

b The 95% confidence interval for $\mu_1 - \mu_2 = \mu_d$ is

$$\bar{d} \pm t_{.025} \frac{s_d}{\sqrt{n}} \Rightarrow -1.58 \pm 2.776 \frac{.8167}{\sqrt{5}} \Rightarrow -1.58 \pm 1.014$$

or $-2.594 < (\mu_1 - \mu_2) < -.566$.

c The inequality to be solved is

$$t_{\alpha/2} SE \leq B$$

We need to estimate the difference in mean temperatures between ground-based and air-based sensors to within .2 degrees centigrade with 95% confidence. Since this is a paired experiment, the inequality becomes

$$t_{.025} \frac{s_d}{\sqrt{n}} \leq .2$$

With $s_d = .8167$ and *n* represents the number of pairs of observations, consider the sample size obtained by replacing $t_{.025}$ by $z_{.025} = 1.96$.

$$1.96 \frac{.8167}{\sqrt{n}} \leq .2$$

$$\sqrt{n} \geq 8.0019 \quad \Rightarrow \quad n = 64.03 \quad \text{or} \quad n = 65$$

Since the value of *n* is greater than 30, the use of $z_{\alpha/2}$ for $t_{\alpha/2}$ is justified.

10.44 A paired-difference test is used, since the two samples are not random and independent (within any sample, the dye 1 and dye 2 measurements are related). The hypothesis of interest is

$$H_0 : \mu_1 - \mu_2 = 0 \quad H_a : \mu_1 - \mu_2 \neq 0$$

The table of differences, along with the calculation of $\bar{d}$ and s_d^2, is presented below.

Sample	1	2	3	4	5	6	7	8	9	Total
d_i	2	1	–1	2	3	–1	0	2	2	10

$$\bar{d} = \frac{\sum d_i}{n} = \frac{10}{9} = 1.11$$

$$s_d^2 = \frac{\sum d_i^2 - \frac{\left(\sum d_i\right)^2}{n}}{n-1} = \frac{28 - \frac{(10)^2}{9}}{8} = 2.1111 \quad \text{and} \quad s_d = 1.452966$$

and the test statistic is

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{1.11 - 0}{\frac{1.452966}{\sqrt{9}}} = 2.29$$

A rejection region with $\alpha = .05$ and $df = n - 1 = 8$ is $|t| > t_{.025} = 2.306$, and H_0 is not rejected at the 5% level of significance. We cannot conclude that there is a difference in the mean brightness scores.

14: Analysis of Categorical Data

14.1 See Section 14.1 of the text.

14.2 Index Table 5, Appendix I, with χ^2_α and the appropriate degrees of freedom.

a $\chi^2_{.05} = 7.81$ **b** $\chi^2_{.01} = 20.09$

c $\chi^2_{.005} = 32.8013$ **d** $\chi^2_{.01} = 24.725$

14.3 For a test of specified cell probabilities, the degrees of freedom are $k-1$. Use Table 5, Appendix I:

a $df = 6$; $\chi^2_{.05} = 12.59$; reject H_0 if $X^2 > 12.59$

b $df = 9$; $\chi^2_{.01} = 21.666$; reject H_0 if $X^2 > 21.666$

c $df = 13$; $\chi^2_{.005} = 29.814$; reject H_0 if $X^2 > 29.8194$

d $df = 2$; $\chi^2_{.05} = 5.99$; reject H_0 if $X^2 > 5.99$

14.5 **a** Three hundred responses were each classified into one of five categories. The objective is to determine whether or not one category is preferred over another. To see if the five categories are equally likely to occur, the hypothesis of interest is

$$H_0 : p_1 = p_2 = p_3 = p_4 = p_5 = \frac{1}{5}$$

versus the alternative that at least one of the cell probabilities is different from 1/5.

b The number of degrees of freedom is equal to the number of cells, *k*, less one degree of freedom for each linearly independent restriction placed on $p_1, p_2, \ldots, p_k$. For this exercise, $k = 5$ and one degree of freedom is lost because of the restriction that

$$\sum p_i = 1$$

Hence, X^2 has $k-1 = 4$ degrees of freedom.

c The rejection region for this test is located in the upper tail of the chi-square distribution with $df = 4$. From Table 5, the appropriate upper-tailed rejection region is $X^2 > \chi^2_{.05} = 9.4877$.

d The test statistic is

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

which, when *n* is large, possesses an approximate chi-square distribution in repeated sampling. The values of O_i are the actual counts *observed* in the experiment, and

$$E_i = np_i = 300(1/5) = 60 .$$

A table of observed and expected cell counts follows:

Category	1	2	3	4	5
O_i	47	63	74	51	65
E_i	60	60	60	60	60

Then

$$X^2 = \frac{(47-60)^2}{60} + \frac{(63-60)^2}{60} + \frac{(74-60)^2}{60} + \frac{(51-60)^2}{60} + \frac{(65-60)^2}{60}$$

$$= \frac{480}{60} = 8.00$$

e Since the observed value of X^2 does not fall in the rejection region, we cannot conclude that there is a difference in the preference for the five categories.

14.9 If the frequency of occurrence of a heart attack is the same for each day of the week, then when a heart attack occurs, the probability that it falls in one cell (day) is the same as for any other cell (day). Hence,

$$H_0: p_1 = p_2 = \cdots = p_7 = \frac{1}{7}$$

vs. H_a : at least one p_i is different from the others , or equivalently,

$$H_a: p_i \neq p_j \text{ for some pair } i \neq j$$

Since $n = 200$, $E_i = np_i = 200(1/7) = 28.571429$ and the test statistic is

$$X^2 = \frac{(24-28.571429)^2}{28.571429} + \cdots + \frac{(29-28.571429)^2}{28.571429} = \frac{103.71429}{28.571429} = 3.63$$

The degrees of freedom for this test of specified cell probabilities is $k-1 = 7-1 = 6$ and the upper tailed rejection region is

$$X^2 > \chi^2_{.05} = 12.59$$

H_0 is not rejected. There is insufficient evidence to indicate a difference in frequency of occurrence from day to day.

14.10 It is necessary to determine whether proportions at a given hospital differ from the population proportions. A table of observed and expected cell counts follows:

Disease	A	B	C	D	Other	Totals
O_i	43	76	85	21	83	308
E_i	46.2	64.68	55.44	43.12	98.56	308

The null hypothesis to be tested is

$$H_0: p_1 = .15;\ p_2 = .21;\ p_3 = .18;\ p_4 = .14$$

against the alternative that at least one of these probabilities is incorrect. The test statistic is

$$X^2 = \frac{(43-46.2)^2}{46.2} + \frac{(76-64.88)^2}{64.88} + \cdots + \frac{(83-98.56)^2}{98.56} = 31.77$$

The number of degrees of freedom is $k-1=4$ and, since the observed value of $X^2 = 31.77$ is greater than $\chi^2_{.005}$, the *p*-value is less than .005 and the results are declared highly significant. We reject H_0 and conclude that the proportions of people dying of diseases A, B, C, and D at this hospital differ from the proportions for the larger population.

14.11 Similar to previous exercises. The hypothesis to be tested is

$$H_0: p_1 = p_2 = \cdots = p_{12} = \frac{1}{12}$$

versus H_a : at least one p_i is different from the others

with

$$E_i = np_i = 400(1/12) = 33.333.$$

The test statistic is

$$X^2 = \frac{(38-33.33)^2}{33.33} + \cdots + \frac{(35-33.33)^2}{33.33} = 13.58$$

The upper tailed rejection region is with $\alpha = .05$ and $k-1 = 11$ *df* is $X^2 > \chi^2_{.05} = 19.675$. The null hypothesis is not rejected and we cannot conclude that the proportion of cases varies from month to month.

14.16 **a** The experiment is analyzed as a 3×4 contingency table. Hence, the expected cell counts must be obtained for each of the cells. Since values for the cell probabilities are not specified by the null hypothesis, they must be estimated, and the appropriate estimator is

$$\hat{E}_{ij} = \frac{r_i c_j}{n},$$

where r_i is the total for row *i* and c_j is the total for column *j* (see Section 14.4). The contingency table, including column and row totals and the estimated expected cell counts (in parentheses) follows.

	Column				
Row	1	2	3	4	**Total**
1	120 (67.68)	70 (66.79)	55 (67.97)	16 (58.56)	261
2	79 (84.27)	108 (83.17)	95 (84.64)	43 (72.91)	325
3	31 (78.05)	49 (77.03)	81 (78.39)	140 (67.53)	301
Total	230	227	231	199	887

The test statistic is

$$X^2 = \sum \frac{\left(O_{ij} - \hat{E}_{ij}\right)^2}{\hat{E}_{ij}} = \frac{(120-67.68)^2}{67.68} + \cdots + \frac{(140-67.53)^2}{67.53} = 211.71$$

using the two-decimal accuracy given above. The degrees of freedom are

$$df = (r-1)(c-1) = (3-1)(4-1) = 6$$

b Similar to part **a**. The estimated expected cell counts are calculated as $\hat{E}_{ij} = \frac{r_i c_j}{n}$, and are shown in parentheses in the table below.

	Column			
Row	1	2	3	**Total**
1	35 (37.84)	16 (26.37)	84 (70.80)	135
2	120 (117.16)	92 (81.63)	206 (219.20)	418
Total	155	108	290	553

The test statistic is calculated (using calculator accuracy rather than the two-decimal accuracy given in part (a) as

$$X^2 = \frac{(35-37.84)^2}{37.84} + \cdots + \frac{(206-219.20)^2}{219.20} = 8.93$$

The degrees of freedom are $(r-1)(c-1) = (2-1)(3-1) = 2$.

14.18 **a** Since $r = 2$ and $c = 3$, the total degrees of freedom are $(r-1)(c-1) = (1)(2) = 2$.

b The experiment is analyzed as a 2×3 contingency table. The contingency table, including column and row totals and the estimated expected cell counts, follows.

	Column			
Row	1	2	3	**Total**
1	37 (42.23)	34 (37.31)	93 (84.46)	164
2	66 (60.77)	57 (53.69)	113 (121.54)	236
Total	103	91	206	400

The estimated expected cell counts were calculated as:

$$\hat{E}_{11} = \frac{r_1c_1}{n} = \frac{164(103)}{400} = 42.23$$

$$\hat{E}_{12} = \frac{r_1c_2}{n} = \frac{164(91)}{400} = 37.31 \quad \text{and so on.}$$

Then

$$X^2 = \frac{(37-42.23)^2}{42.23} + \frac{(34-37.31)^2}{37.31} + \cdots + \frac{(113-121.54)^2}{121.54} = 3.059 .$$

c With $\alpha = .01$, a one-tailed rejection region is found using Table 5 to be $X^2 > \chi^2_{.01} = 9.21$.

d-e The observed value of $X^2 = 3.059$ does not fall in the rejection region. Hence, H_0 is not rejected. There is no reason to expect a dependence between rows and columns. In fact, $X^2 = 3.059$ has a p-value $> .10$.

14.20 **a** The hypothesis to be tested is

H_0 : opinion is independent of political affiliation

H_a : opinion is dependent on political affiliation

and the ***Minitab*** printout below shows the observed and estimated expected cell counts.

Chi-Square Test: Support, Oppose, Unsure

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

      Support  Oppose  Unsure  Total
    1     256     163      22    441
       236.60  182.50   21.90
        1.590   2.083   0.000

    2      60      40       5    105
        56.33   43.45    5.21
        0.239   0.274   0.009

    3     235     222      24    481
       258.06  199.05   23.89
        2.061   2.646   0.001

Total     551     425      51   1027
Chi-Sq = 8.903, DF = 4, P-Value = 0.064
```

The test statistic is the chi-square statistic given in the printout as $X^2 = 8.903$ with p-value $= .064$. Since the p-value is greater than .05, H_0 is not rejected. There is insufficient evidence to indicate that there is a difference in a person's opinion depending on the political party with which he is affiliated.

b Even though there are no significant differences, we can still look at the conditional distributions of opinions for the three groups, shown in the table below.

	Support	**Oppose**	**Unsure**
Democrats	$\frac{256}{441} = .58$	$\frac{163}{441} = .37$	$\frac{22}{441} = .05$
Independents	$\frac{60}{105} = .57$	$\frac{40}{105} = .38$	$\frac{5}{105} = .05$
Republicans	$\frac{235}{481} = .49$	$\frac{222}{481} = .46$	$\frac{24}{481} = .05$

You can see that Democrats and Independents have almost identical opinions on mandatory healthcare, while Republicans are less likely to be supportive.

14.21 **a** The hypothesis of independence between attachment pattern and child care time is tested using the chi-square statistic. The contingency table, including column and row totals and the estimated expected cell counts, follows.

	Child Care			
Attachment	Low	Moderate	High	**Total**
Secure	24 (24.09)	35 (30.97)	5 (8.95)	64
Anxious	11 (10.91)	10 (14.03)	8 (4.05)	29
Total	111	51	297	459

The test statistic is

$$X^2 = \frac{(24-24.09)^2}{24.09} + \frac{(35-30.97)^2}{30.97} + \cdots + \frac{(8-4.05)^2}{4.05} = 7.267$$

and the rejection region is $X^2 > \chi^2_{.05} = 5.99$ with 2 *df*. H_0 is rejected. There is evidence of a dependence between attachment pattern and child care time.

b The value $X^2 = 7.267$ is between $\chi^2_{.05}$ and $\chi^2_{.025}$ so that $.025 < p\text{-value} < .05$. The results are significant.

14.24 **a** The hypothesis of independence between opinion and political affiliation is tested using the chi-square statistic. The contingency table, including column and row totals and the estimated expected cell counts, follows.

	Opinion			
Political Affiliation	Know all facts	Cover up	Not sure	**Total**
Democrats	42 (53.48)	309 (284.38)	31 (44.14)	382
Republicans	64 (49.84)	246 (265.02)	46 (41.14)	356
Independents	20 (22.68)	115 (120.60)	27 (18.72)	162
Total	126	670	104	900

The test statistic is

$$X^2 = \frac{(42-53.48)^2}{53.48} + \frac{(309-284.38)^2}{284.38} + \cdots + \frac{(27-18.72)^2}{18.72} = 18.711$$

The test statistic is very large, compared to the largest value in Table 5 $\left(\chi^2_{.005} = 14.86\right)$, so that $p\text{-value} < .005$ and H_0 is rejected. There is evidence of a difference in the distribution of opinions depending on political affiliation.

b You can see that the percentage of Democrats who think there was a cover-up is higher than the same percentages for either Republicans or Independents.

14.29 Because a set number of Americans in each sub-population were each fixed at 200, we have a contingency table with fixed rows. The table, with estimated expected cell counts appearing in parentheses, follows.

	Yes	**No**	**Total**
White-American	40 (62)	160 (138)	200
African-American	56 (62)	144 (138)	200
Hispanic-American	68 (62)	132 (138)	200
Asian-American	84 (62)	116 (138)	200
Total	248	552	800

The test statistic is

$$X^2 = \frac{(40-62)^2}{62} + \frac{(56-62)^2}{62} + \cdots + \frac{(116-138)^2}{138} = 24.31$$

and the rejection region with 3 *df* is $X^2 > 11.3449$. H_0 is rejected and we conclude that the incidence of parental support is dependent on the sub-population of Americans.

14.34 **a** The number of observations per column were selected prior to the experiment. The test procedure is identical to that used for an $r \times c$ contingency table. The contingency table generated by ***Minitab***, including column and row totals and the estimated expected cell counts, follows.

Chi-Square Test: 20s, 30s, 40s, 50s, 60+

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts
         20s    30s    40s    50s    60+  Total
    1     31     42     47     48     53    221
       44.20  44.20  44.20  44.20  44.20
       3.942  0.110  0.177  0.327  1.752

    2     69     58     53     52     47    279
       55.80  55.80  55.80  55.80  55.80
       3.123  0.087  0.141  0.259  1.388
Total    100    100    100    100    100    500
Chi-Sq = 11.304, DF = 4, P-Value = 0.023
```

The observed value of the test statistic is $X^2 = 11.304$ with *p*-value = .023 and the null hypothesis is rejected at the 5% level of significance. There is sufficient evidence to indicate that the proportion of adults who attend church regularly differs depending on age.

b The percentage who attend church increases with age.

14.36 To test for homogeneity of the four binomial populations, we use chi-square statistic and the 2×4 contingency table shown below.

	Rhode Island	**Colorado**	**California**	**Florida**	**Total**
Participate	46 (63.32)	63 (78.63)	108 (97.88)	121 (97.88)	338
Do not participate	149 (131.38)	178 (162.37)	192 (202.12)	179 (202.12)	698
Total	195	241	300	300	1036

The test statistic is $$X^2 = \frac{(46-63.62)^2}{63.62} + \frac{(63-78.63)^2}{78.63} + \cdots + \frac{(179-202.12)^2}{202.12} = 21.51$$

With $df = 3$, the *p*-value is less than .005 and H_0 is rejected. There is a difference in the proportions for the four states. The difference can be seen by considering the proportion of people participating in each of the four states:

	Rhode Island	**Colorado**	**California**	**Florida**
Participate	$\frac{46}{195} = .24$	$\frac{63}{241} = .26$	$\frac{108}{300} = .36$	$\frac{121}{300} = .40$

14.45 **a** Similar to previous exercises. The contingency table, including column and row totals and the estimated expected cell counts, follows.

Condition	Treated	Untreated	**Total**
Improved	117 (95.5)	74 (95.5)	191
Not improved	83 (104.5)	126 (104.5)	209
Total	200	200	400

The test statistic is

$$X^2 = \frac{(117-95.5)^2}{95.5} + \frac{(74-95.5)^2}{95.5} + \cdots + \frac{(126-104.5)^2}{104.5} = 18.527$$

To test a one-tailed alternative of "effectiveness", first check to see that $\hat{p}_1 > \hat{p}_2$. Then the rejection region with 1 *df* has a right-tail area of $2(.05) = .10$ or $X^2 > \chi^2_{2(.05)} = 2.706$. H_0 is rejected and we conclude that the serum is effective.

b Consider the treated and untreated patients as comprising random samples of two hundred each, drawn from two populations (i.e., a sample of 200 treated patients and a sample of 200 untreated patients). Let p_1 be the probability that a treated patient improves and let p_2 be the probability that an untreated patient improves. Then the hypothesis to be tested is

$$H_0: p_1 - p_2 = 0 \qquad H_a: p_1 - p_2 > 0$$

Using the procedure described in Chapter 9 for testing an hypothesis about the difference between two binomial parameters, the following estimators are calculated:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{117}{200} \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{74}{200} \qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{117+74}{400} = .4775$$

The test statistic is

$$z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}\hat{q}(1/n_1 + 1/n_2)}} = \frac{.215}{\sqrt{.4775(.5225)(.01)}} = 4.304$$

And the rejection region for $\alpha = .05$ is $z > 1.645$. Again, the test statistic falls in the rejection region. We reject the null hypothesis of no difference and conclude that the serum is effective. Notice that

$$z^2 = (4.304)^2 = 18.52 = X^2 \text{ (to within rounding error)}$$

14.47 Refer to Section 9.6. The two-tailed z test was used to test the hypothesis

$$H_0: p_1 - p_2 = 0 \qquad H_a: p_1 - p_2 \neq 0$$

using the test statistic

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\Rightarrow \quad z = \frac{(\hat{p}_1 - \hat{p}_2)^2}{\hat{p}\hat{q}\left(\frac{n_1+n_2}{n_1 n_2}\right)} = \frac{n_1 n_2 (\hat{p}_1 - \hat{p}_2)^2}{\hat{p}\hat{q}(n_1+n_2)}$$

Note that

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

Now consider the chi-square test statistic used in Exercise 14.45. The hypothesis to be tested is

H_0 : independence of classification $\qquad$ H_a : dependence of classification

That is, the null hypothesis asserts that the percentage of patients who show improvement is independent of whether or not they have been treated with the serum. If the null hypothesis is true, then $p_1 = p_2$. Hence, the two tests are designed to test the same hypothesis. In order to show that z^2 is equivalent to X^2, it is necessary to rewrite the chi-square test statistic in terms of the quantities, $\hat{p}_1, \hat{p}_2, \hat{p}, n_1$ and n_2.

1 Consider O_{11}, the observed number of treated patients who have improved. Since $\hat{p}_1 = O_{11}/n_1$, we have $O_{11} = n_1\hat{p}_1$. Similarly,

$$O_{21} = n_1\hat{q}_1 \qquad O_{12} = n_2\hat{p}_2 \qquad O_{22} = n_2\hat{q}_2$$

2 The estimated expected cell counts are calculated under the assumption that the null hypothesis is true. Consider

$$E_{11} = \frac{r_1c_1}{n} = \frac{(O_{11}+O_{12})(O_{11}+O_{21})}{n_1+n_2} = \frac{(x_1+x_2)(O_{11}+O_{21})}{n_1+n_2} = n_1\hat{p}$$

Similarly,

$$\hat{E}_{21} = n_1\hat{q} \qquad \hat{E}_{12} = n_2\hat{p} \qquad \hat{E}_{22} = n_2\hat{q}$$

The table of observed and estimated expected cell counts follows.

	Treated	**Untreated**	**Total**
Improved	$n_1\hat{p}_1$ $(n_1\hat{p})$	$n_2\hat{p}_2$ $(n_2\hat{p})$	x_1+x_2
Not improved	$n_1\hat{q}_1$ $(n_1\hat{q})$	$n_2\hat{q}_1$ $(n_2\hat{q})$	$n-(x_1+x_2)$
Total	n_1	n_2	n

Then

$$\begin{aligned}
X^2 &= \sum \frac{(O_{ij}-E_{ij})^2}{E_{ij}} \\
&= \frac{n_1^2(\hat{p}_1-\hat{p})^2}{n_1\hat{p}} + \frac{n_1^2(\hat{q}_1-\hat{q})^2}{n_1\hat{q}} + \frac{n_2^2(\hat{p}_2-\hat{p})^2}{n_2\hat{p}} + \frac{n_2^2(\hat{q}_2-\hat{q})^2}{n_2\hat{q}} \\
&= \frac{n_1(\hat{p}_1-\hat{p})^2}{\hat{p}} + \frac{n_1[(1-\hat{p}_1)-(1-\hat{p})]^2}{\hat{q}} + \frac{n_2(\hat{p}_2-\hat{p})^2}{\hat{p}} + \frac{n_2[(1-\hat{p}_2)-(1-\hat{p})]^2}{\hat{q}} \\
&= \frac{(1-\hat{p}_1)n_1(\hat{p}_1-\hat{p})^2 + n_1\hat{p}(\hat{p}_1-\hat{p})^2}{\hat{p}\hat{q}} + \frac{(1-\hat{p}_2)n_2(\hat{p}_2-\hat{p})^2 + n_2\hat{p}(\hat{p}_2-\hat{p})^2}{\hat{p}\hat{q}} \\
&= \frac{n_1(\hat{p}_1-\hat{p})^2}{\hat{p}\hat{q}} + \frac{n_2(\hat{p}_2-\hat{p})^2}{\hat{p}\hat{q}}
\end{aligned}$$

Substituting for $\hat{p}$, we obtain

$$\begin{aligned}
X^2 &= \frac{n_1}{\hat{p}\hat{q}}\left[\frac{n_1\hat{p}_1+n_2\hat{p}_1-n_1\hat{p}_1-n_2\hat{p}_2}{n_1+n_2}\right]^2 + \frac{n_2}{\hat{p}\hat{q}}\left[\frac{n_1\hat{p}_2+n_2\hat{p}_2-n_1\hat{p}_1-n_2\hat{p}_2}{n_1+n_2}\right]^2 \\
&= \frac{n_1n_2^2(\hat{p}_1-\hat{p}_2)^2 + n_1^2n_2(\hat{p}_1-\hat{p}_2)^2}{\hat{p}\hat{q}(n_1+n_2)^2} = \frac{n_1n_2(\hat{p}_1-\hat{p}_2)^2}{\hat{p}\hat{q}(n_1+n_2)}
\end{aligned}$$

Note that X^2 is identical to z^2, as defined at the beginning of the exercise.

14.53 **a** The contingency table with estimated expected cell counts in parentheses is shown in the ***Minitab*** printout below.

Chi-Square Test: Excellent, Good, Fair

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts
     Excellent   Good   Fair  Total
    1       43     48      9    100
         43.50  50.50   6.00
         0.006  0.124  1.500
```

```
    2         44     53      3    100
          43.50  50.50   6.00
          0.006  0.124  1.500

Total         87    101     12    200

Chi-Sq = 3.259, DF = 2, P-Value = 0.196
```

The test statistic is $$X^2 = \frac{(43-43.5)^2}{43.5} + \frac{(48-50.5)^2}{50.5} + \cdots + \frac{(3-6.00)^2}{6.00} = 3.259$$

The observed value of X^2 is less than $\chi^2_{.10}$ so that $p\text{-value} > .10$ (the exact $p\text{-value} = .196$ from the printout) and H_0 is not rejected. There is no evidence of a difference due to gender.

b The contingency table with estimated expected cell counts in parentheses is shown in the ***Minitab*** printout below.

Chi-Square Test: Excellent, Good, Fair, Poor

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts
       Excellent   Good   Fair   Poor  Total
    1          4     42     41     13    100
            3.50  45.00  38.00  13.50
           0.071  0.200  0.237  0.019

    2          3     48     35     14    100
            3.50  45.00  38.00  13.50
           0.071  0.200  0.237  0.019
Total          7     90     76     27    200
Chi-Sq = 1.054, DF = 3, P-Value = 0.788
2 cells with expected counts less than 5.
```

The test statistic is

$$X^2 = \frac{(4-3.5)^2}{3.5} + \frac{(42-45)^2}{45} + \cdots + \frac{(14-13.5)^2}{13.5} = 1.054$$

The observed value of X^2 is less than $\chi^2_{.10}$ so that $p\text{-value} > .10$ (the exact $p\text{-value} = .788$ from the printout) and H_0 is not rejected. There is no evidence of a difference due to gender.

c Notice that the computer printout in part **b** warns that 2 cells have expected cell counts less than 5. This is a violation of the assumptions necessary for this test, and results should thus be viewed with caution.

14.59 **a** The 2×3 contingency table is analyzed as in previous exercises. The ***Minitab*** printout below shows the observed and estimated expected cell counts, the test statistic and its associated *p*-value.

Chi-Square Test: 3 or fewer, 4 or 5, 6 or more

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts
        3 or
       fewer  4 or 5  6 or more  Total
    1     49      43         34    126
       37.89   42.63      45.47
       3.254   0.003      2.895

    2     31      47         62    140
       42.11   47.37      50.53
       2.929   0.003      2.605

Total     80      90         96    266
Chi-Sq = 11.690, DF = 2, P-Value = 0.003
```

The results are highly significant ($p\text{-value} = .003$) and we conclude that there is a difference in the susceptibility to colds depending on the number of relationships you have.

b The proportion of people with colds is calculated conditionally for each of the three groups, and is shown in the table below.

	Three or fewer	**Four or five**	**Six or more**

Cold	$\frac{49}{80}=.61$	$\frac{43}{90}=.48$	$\frac{34}{96}=.35$
No cold	$\frac{31}{80}=.39$	$\frac{47}{90}=.52$	$\frac{62}{96}=.65$
Total	1.00	100	1.00

As the researcher suspects, the susceptibility to a cold seems to decrease as the number of relationships increases!

14.61 The null hypothesis to be tested is

$$H_0: p_1=\frac{1}{8};\ p_2=\frac{1}{8};\ p_3=\frac{1}{8};\ p_4=\frac{1}{8};\ p_5=\frac{2}{8};\ p_6=\frac{2}{8}$$

against the alternative that at least one of these probabilities is incorrect. A table of observed and expected cell counts follows:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
O_i	95	110	125	75	181	214
E_i	100	100	100	100	200	200

The test statistic is

$$X^2=\frac{(95-100)^2}{100}+\frac{(110-100)^2}{100}+\cdots+\frac{(214-200)^2}{200}=16.535$$

The number of degrees of freedom is $k-1=5$ and the rejection region with $\alpha=.05$ is $X^2>\chi^2_{.05}=11.07$ and H_0 is rejected. The manager's claim is refuted.